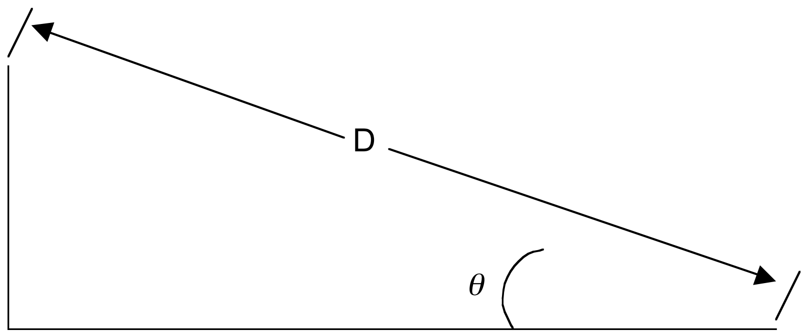
MOMENT OF INERTIA

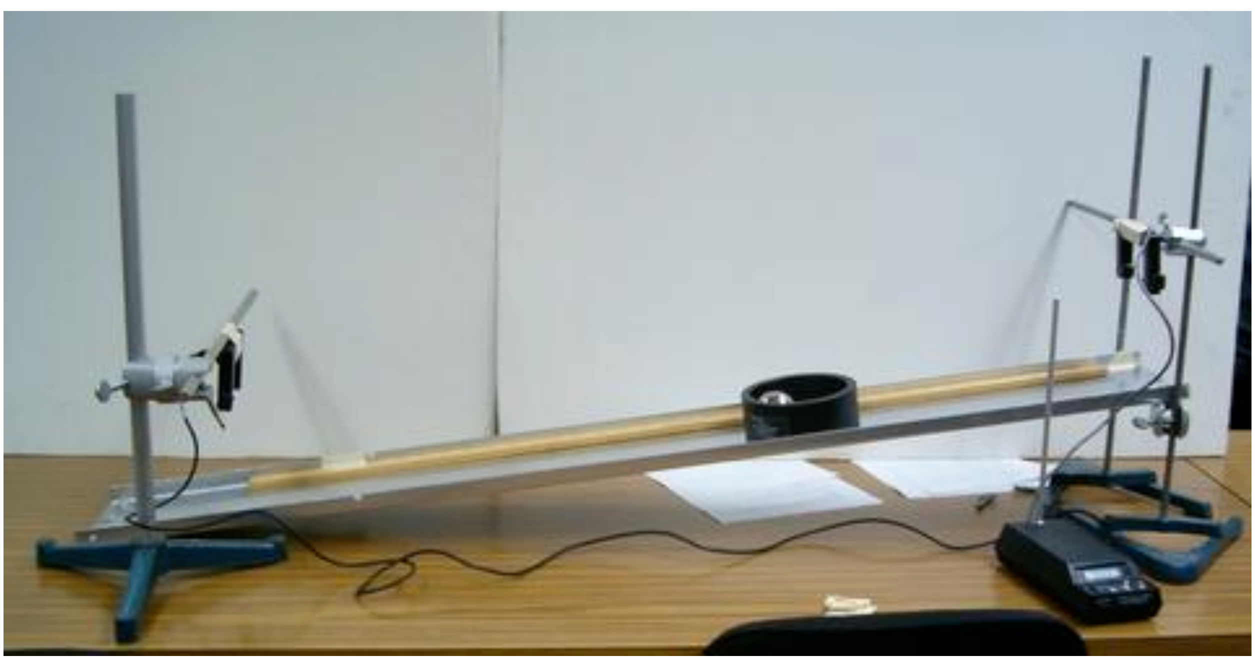
Saddleback College Physics Department

Purpose

Given a hollow cylinder and either a solid sphere **or** a solid cylinder, measure the time it takes each to roll down an inclined plane and compare that time with the theoretical time for each object to go down the plane at two different angles.



The photographs below show <u>one</u> example of how the lab could be set up. Keep in mind that there are many comparable ways to set up this apparatus that may be less work-intensive. There is no need to have a meter stick on the ramp as shown in this photo and you should secure your ramp in place using tape so that it does not slide.



Theory

The moment of inertia of the three objects are found to be

$$I_{\substack{solid\ sphere \\ (center)}} = \frac{2}{5}MR^2$$

$$I_{\substack{solid\ cylinder \\ (center)}} = \frac{1}{2}MR^2$$

$$I_{\substack{thin\ walled\ hollow\ cylinder \\ (center)}} = MR^2$$

Note that each of these can be expressed as $I = CMR^2$ with C being 2/5 for the sphere, 1/2 for the solid cylinder and 1 for the hollow cylinder.

Use conservation of energy to find the final speed of the objects at the bottom of the ramp, then use the equation for average speed (with a constant velocity) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{v_\circ + v_f}{2}$ to derive the following equation:

$$t_{theoretical} = \sqrt{\frac{2(1+C)D}{g\sin\theta}}$$

For ANALYSIS part (4) ONLY: The equation for the moment of inertia of a hollow cylinder, with inner radius R_1 and outer radius R_2 and mass M is given by $I = \frac{M(R_1{}^2 + R_2{}^2)}{2}$

Equipment

Smooth incline, photogates, meter stick, plumb bob, calipers, hoop, sphere and/or disk

Procedure

DO NOT DISSASEMBLE YOUR SET-UP UNTIL YOUR CALCULATIONS ARE COMPLETED.

1) Adjust the angle of the incline to approximately 5 degrees using the angle meter. Then calculate the angle by measuring the vertical and horizontal heights (corresponding to the distance D) using the plumb bob and meter stick, then calculate the angle with trigonometry. You decide which one is most accurate and should be used in your calculations!
2) Measure the distance, D, that each object rolls down the incline. Check your measured value for D against the D value calculated from the Pythagorean theorem using the horizontal and vertical heights. You decide which one is most accurate and should be used in your calculations!
3) Measure the time for at least ten trials for each object, using the photogates. It may be easiest to use the pendulum mode on the photogate when timing the hoop since it can then break the beam three times and will only start and stop with the first and third break of the beam.
4) Measure the inside and outside radius for the hollow cylinder to use in ANALYSIS part (4)
5) Repeat all of the above steps for an angle of inclination of approximately 10 degrees.

Analysis

1) Calculate an average time [and if your instructor specifies, calculate a standard deviation for each object].
2) Calculate a theoretical time [if your instructor specifies, calculate an estimated percent error for the theoretical time for each object at each given angle].
3) Compare the results (is there agreement between the theoretical and measured times?).
4) Show whether the equation for the moment of inertia of a hollow cylinder, with inner radius R_1 and outer radius R_2, $I = \frac{M(R_1^2 + R_2^2)}{2}$ yields more accurate results when used to calculate the theoretical times for the hollow cylinder. (**Important Note:** You will need to start all over with your calculations, using conservation of mechanical energy, with the "I"given above. You cannot write this "I" in terms of C! When you plug in $\omega_f = \frac{v_{f_{CM}}}{R}$, you can use $R = R_2$. Show that the new time is given by: $\Delta t = \frac{1}{R_2}\sqrt{\frac{D(3R_2^2 + R_1^2)}{g \sin\theta}}$)

In your conclusion, be sure to summarize all of your results in a summary table showing all of the measured average times and all of the theoretical times as well as the respective % differences.