The Density of a Metal Cylinder

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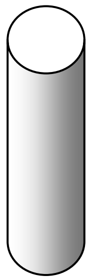
PH 4A

Date & Time of Lab: 1/22/03, 3:00 - 6:00 p.m.

Purpose: To determine the density of a metal cylinder from measurements of the cylinder's height, diameter and mass, then compare this value to the handbook value for that metal's density.

Sketch:



Equipment: Calipers, Metal Cylinder and Balance Scale

Theory: The average diameter, $\bar{d}$, and the average height, $\bar{h}$, of the cylinder are calculated using the average equation below for *n* measurements:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \quad (1) \qquad \text{(where } \bar{x} = \bar{d} \ \& \ \bar{x} = \bar{h}\text{)}$$

The average volume of the cylinder can be calculated using the average radius, $\bar{r}$, of the cylinder (where $\bar{r} = \frac{\bar{d}}{2}$) and the average height, $\bar{h}$, of the cylinder as seen in the equation below:

$$\bar{V} = \pi \bar{r}^2 \bar{h} \qquad (2)$$

The density of the cylinder is calculated using the equation below:

$$\rho_{\exp} = \frac{\bar{m}}{\bar{V}} \qquad (3)$$

where $\bar{m} = average\ \ cylinder\ \ mass$
$\bar{V} = average\ \ cylinder\ \ volume$

The percent difference between the handbook and experimental densities of the metal cylinder are found using the equation below.

$$\%Difference = \frac{\left|Value_{theoretical} - Value_{experimental}\right|}{V_{theoretical}} \times 100 \qquad (4)$$

The absolute error in the experimental density of the metal cylinder is found using the exact differential below:

$$\delta\rho_{\exp} = \sqrt{\left[\left(\frac{\partial\rho_{\exp}}{\partial\bar{r}}\right)\delta\bar{r}\right]^2 + \left[\left(\frac{\partial\rho_{\exp}}{\partial\bar{h}}\right)\delta\bar{h}\right]^2 + \left[\left(\frac{\partial\rho_{\exp}}{\partial\bar{m}}\right)\delta\bar{m}\right]^2} \quad (5)$$

where $\delta\bar{\rho}_{\exp}$ = random error in experimental density of cylinder
$\delta\bar{r}$ = random error in radius of cylinder
$\delta\bar{h}$ = random error in height of cylinder
$\delta\bar{m}$ = random error in mass of cylinder

The Percent Error in the experimental density was found using the equation below:

$$\%\,Error = \frac{\delta\rho_{\exp}}{\rho_{\exp}} \times 100 \quad (6)$$

Procedure:

1. Record the type of metal cylinder you are using (either brass or copper).
2. Use the calipers to measure the height of the cylinder 10 times and the diameter of the cylinder ten times, making sure to take each measurement at a different place on the cylinder. Record the 20 measurements in a table and calculate their averages.
3. Measure the mass of the cylinder 5 times using the balance scale. Record the measurements in a table and calculate their average.

Data:

Measurement #	1	2	3	4	5	6	7	8	9	10	Average	Estimated random error
Diameter (mm)	12.66	12.66	12.66	12.66	12.70	12.70	12.60	12.66	12.66	12.60	12.656	---------
Radius (mm)	6.33	6.33	6.33	6.33	6.35	6.35	6.30	6.33	6.33	6.30	6.325	$\delta\bar{r} = \pm 0.01$
Height (mm)	50.85	50.70	50.65	50.70	50.75	50.75	50.75	50.85	50.83	50.80	50.763	$\delta\bar{h} = \pm 0.01$
Mass (g)	57.43	57.41	57.34	57.39	57.41						57.396	$\delta\bar{m} = \pm 0.15$

$\rho_{theo} = 8.900\ {}^{g}/_{cm^3}$

Analysis/Calculations:

Using equation number (1) from the theory, the average radius, height and mass of the metal cylinder are calculated. A sample calculation is shown below.

$$\bar{r} = \frac{\sum_{i=1}^{n} r_i}{n} = \frac{(6.33+6.33+6.33+6.33+6.35+6.35+6.30+6.33+6.33+6.30)\text{mm}}{10}$$

The experimental density of the cylinder is found from substituting equation (2) into equation (3) to yield the sample calculation below:

$$\rho_{\exp} = \frac{\overline{m}}{\overline{V}}$$

$$\rho_{\exp} = \frac{\overline{m}}{\pi \overline{r}^2 \overline{h}}$$

$$\rho_{\exp} = \frac{57.396\ g}{\pi \cdot (6.325\ mm)^2 (50.763\ mm)}$$

$$\rho_{\exp} = \frac{57.396\ g}{6.380\ \text{cm}^3}$$

$$\rho_{\exp} = 9.00\ {}^{g}\!/_{cm^3}$$

The percent difference between the experimental and theoretical densities of the yellow brass cylinder are found using equation (4) as seen below:

$$\%difference = \frac{\left| 9.00\ {}^{g}\!/_{cm^3} - 8.900\ {}^{g}\!/_{cm^3} \right|}{8.900\ {}^{g}\!/_{cm^3}} \times 100 = 1.12\% = 1.1\%$$

Finally the absolute error in $\rho_{\exp}$ is calculated using equation (5) in the theory with a sample calculation being shown below:

$$\delta\rho_{\exp} = \sqrt{\left[\left(\frac{\partial \rho_{\exp}}{\partial \overline{r}}\right)\delta\overline{r}\right]^2 + \left[\left(\frac{\partial \rho_{\exp}}{\partial \overline{h}}\right)\delta\overline{h}\right]^2 + \left[\left(\frac{\partial \rho_{\exp}}{\partial \overline{m}}\right)\delta\overline{m}\right]^2}$$

NOTE: Taking the partial derivatives (shown in the square brackets below) DOES NOT need to be shown in a formal lab write-up and is only shown here for instructional purposes.

RULE for PARTIAL DERIVATIVES:

Treat all variables as a constant except for the variable with respect to which you are taking the partial derivative.

$$\frac{\partial \rho_{\exp}}{\partial \overline{r}} = \frac{\partial}{\partial \overline{r}}\rho_{\exp} = \frac{\partial}{\partial \overline{r}}\left(\frac{\overline{m}}{\pi \overline{r}^2 \overline{h}}\right) = \frac{\overline{m}}{\pi \overline{h}}\frac{\partial}{\partial \overline{r}}\left(\frac{1}{\overline{r}^2}\right) = \frac{\overline{m}}{\pi \overline{h}}\frac{\partial}{\partial \overline{r}}(\overline{r}^{-2}) = -2\frac{\overline{m}}{\pi \overline{h}}\overline{r}^{-3} = \frac{-2\overline{m}}{\pi \overline{h}\,\overline{r}^3}$$

$$\frac{\partial \rho_{\exp}}{\partial \overline{h}} = \frac{\partial}{\partial \overline{h}}\rho_{\exp} = \frac{\partial}{\partial \overline{h}}\left(\frac{\overline{m}}{\pi \overline{r}^2 \overline{h}}\right) = \frac{\overline{m}}{\pi \overline{r}^2}\frac{\partial}{\partial \overline{h}}\left(\frac{1}{\overline{h}}\right) = \frac{\overline{m}}{\pi \overline{r}^2}\frac{\partial}{\partial \overline{h}}(\overline{h}^{-1}) = \frac{\overline{m}}{\pi \overline{r}^2}(-1 \cdot h^{-2}) = \frac{-\overline{m}}{\pi \overline{r}^2 \overline{h}^2}$$

$$\frac{\partial \rho_{\exp}}{\partial \overline{m}} = \frac{\partial}{\partial \overline{m}}\rho_{\exp} = \frac{\partial}{\partial \overline{m}}\left(\frac{\overline{m}}{\pi \overline{r}^2 \overline{h}}\right) = \frac{1}{\pi \overline{r}^2 \overline{h}}\frac{\partial}{\partial \overline{m}}(\overline{m}) = \frac{1}{\pi \overline{r}^2 \overline{h}}$$

$$\delta\rho_{\exp} = \sqrt{\left[\left(\frac{\partial\rho_{\exp}}{\partial\bar{r}}\right)\delta\bar{r}\right]^2 + \left[\left(\frac{\partial\rho_{\exp}}{\partial\bar{h}}\right)\delta\bar{h}\right]^2 + \left[\left(\frac{\partial\rho_{\exp}}{\partial\bar{m}}\right)\delta\bar{m}\right]^2}$$ substituting the partial derivatives into this equation

yields $$\delta\rho_{\exp} = \sqrt{\left[\left(\frac{-2\bar{m}}{\pi\bar{h}\bar{r}^3}\right)\delta\bar{r}\right]^2 + \left[\left(\frac{-\bar{m}}{\pi\bar{r}^2\bar{h}^2}\right)\delta\bar{h}\right]^2 + \left[\left(\frac{1}{\pi\bar{r}^2\bar{h}}\right)\delta\bar{m}\right]^2}$$

$$\delta\rho_{\exp} = \sqrt{\left[\left(\frac{-2(57.396g)}{\pi(50.763\text{mm})(6.325\text{mm})^3}\right)(0.01mm)\right]^2 + \left[\left(\frac{-(57.396g)}{\pi(6.325\text{mm})^2(50.763\text{mm})^2}\right)(0.01mm)\right]^2 + \left[\left(\frac{1}{\pi(6.325\text{mm})^2(50.763\text{mm})}\right)(0.15g)\right]^2}$$

$$\delta\rho_{\exp} = \sqrt{8.092\times10^{-10}\, {}^{g}\!/_{cm^3} + 3.141\times10^{-12}\, {}^{g}\!/_{cm^3} + 5.528\times10^{-10}\, {}^{g}\!/_{cm^3}}$$

$$\delta\rho_{\exp} = 1.365\times10^{-9}\, {}^{g}\!/_{cm^3}$$

The percent error in the experimental density is calculated from equation (6) as seen below:

$$\%\ Error\ in\ \rho_{\exp} = \frac{\delta\rho_{\exp}}{\rho_{\exp}}\times100$$

$$\%\ Error\ in\ \rho_{\exp} = \frac{1.365\times10^{-9}\, {}^{g}\!/_{cm^3}}{9.00\, {}^{g}\!/_{cm^3}}\times100 = 1.5\times10^{-8}\%$$

$$\%\ Error\ in\ \rho_{\exp} = 2\times10^{-8}\%$$

Conclusion:

The objective of this lab was clearly met, as the density of a yellow brass cylinder was determined from measurements of the cylinder's dimensions and mass. The experimental density was then compared to the theoretical density of brass. The density of the brass cylinder was experimentally determined to be $\rho_{\exp} = 9.00\, {}^{g}\!/_{cm^3}$, which has a 1.1 percent difference with the theoretical density of $\rho_{theo} = 8.900\ \text{g}/cm^3$. The percent error in $\rho_{\exp}$ was calculated using error propagation to be $2\times10^{-8}\%$. Therefore the percent difference for $\rho_{\exp}$ does not lie within the allowed margin of error ($\pm 2\times10^{-3}\%$) and the experiment was not successful.

The percent difference might be explained by any of the sources of error listed below:

- The irregularities in the cylinder's shape due to scratches and dings in the metal, which cause imprecision in the measurements of the cylinder's radius and height.
- The most significant source of error is probably that the metal may have contained a different mixture of alloys than those we assumed when looking up the theoretical density in the handbook.

SIGN & DATE: