Rational exponents: $2^{\frac{a}{b}} = \sqrt[b]{2^a} = (\sqrt[b]{2})^a$

We can express roots as rational exponents.

$\sqrt{2} = 2^{\frac{1}{2}}$

$\sqrt[3]{x} = x^{\frac{1}{3}}$

$\sqrt[4]{x^3} = x^{\frac{3}{4}}$

One way to remember how to express roots as rational expressions is the phrase POWER over ROOT.

$\sqrt[5]{x^4}$ is written as $x^{\frac{4}{5}}$
since 4 is the power
and 5 is the root.

Example 1: Simplify each expression

A) $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9} = 3$

B) $4^{\frac{3}{2}}$

$4^{\frac{3}{2}}$ means $\sqrt{4^3} = \sqrt{64} = 8$

Note: $4^{\frac{3}{2}} = (\sqrt{4})^3 = (2)^3 = 8$

example 2: Evaluate

A) $8^{\frac{3}{2}} \cdot 8^{\frac{1}{2}}$

$8^{\frac{3}{2}} \cdot 8^{\frac{1}{2}} = 8^{\frac{3}{2}+\frac{1}{2}} = 8^{\frac{4}{2}} = 8^2 = 64$

B) $3^{\frac{1}{4}} \cdot 3^{\frac{3}{4}}$

C) $\dfrac{6^{\frac{1}{2}}}{6^{-\frac{1}{2}}}$

$\dfrac{6^{\frac{1}{2}}}{6^{-\frac{1}{2}}} = 6^{\frac{1}{2}-(-\frac{1}{2})} = 6^{\frac{1}{2}+\frac{1}{2}} = 6^{\frac{2}{2}} = 6^1 = 6$

D) $X^{\frac{1}{3}} \cdot X^{\frac{2}{3}}$

example 3: rewrite each radical expression as an exponential expression. Simplify if possible.

A) $\sqrt[3]{X^2} =$

B) $\sqrt[9]{8^2}$

C) $\sqrt[6]{27^3}$

Example 4: Rewrite each exponential expression as a radical expression. Simplify if possible.

a) $4^{\frac{3}{2}}$

b) $(4x)^{\frac{3}{2}}$

c) $(8x^4)^{\frac{3}{2}}$

d) $(16x^3y^2)^{\frac{1}{3}}$

Rational exponents Practice Problems

Evaluate:

1. $4^{\frac{1}{2}}$

2. $16^{\frac{3}{2}}$

3. $\dfrac{x^{\frac{1}{2}} \cdot x^{\frac{3}{2}}}{x}$

4. $\sqrt[4]{x^3} \cdot \sqrt{x}$