

VARIATION

Direct variation:

If y varies directly as x , then

$$y = kx$$

where k is referred to as the "constant of variation".

If x represents the number of hours you work in a week and y represents the amount of money you get paid per week we would say that y varies directly as x — as x increases, y increases and as x decreases, y also decreases.

Note: $y = kx$ where k represents the hourly salary.

EXAMPLE 1:

Write an equation that relates x and y if y varies directly as x and when $y = 18$, $x = 3$.

Since y varies directly as x , we know

$$y = kx$$

We need to find the constant of variation, k .

When $y = 18$, $x = 3$.

$$y = kx$$

$$\downarrow \qquad \qquad \downarrow$$

$$18 = k(3)$$

Now we solve for k

$$\frac{18}{3} = \frac{3k}{3}$$

$$k = 6$$

So the equation that relates x and y

is $y = 6x$

Inverse variation:

If y varies inversely as x , then

$$y = \frac{k}{x}$$

If x represents how fast you drive to get to work and y represents how much time it takes to get to work, we would say that y varies inversely as x — as x

increases, y decreases and as x decreases, y increases.

EXAMPLE 2:

y varies inversely as x . When x is 10, y is 2. Find x when y is 4.

First we need to write an equation that relates x and y . Since y varies inversely as x , we know

$$y = \frac{k}{x}$$

When $x = 10$, $y = 2$.

$$2 = \frac{k}{10}$$

$$k = 20 \quad (\text{multiply both sides by } 10)$$

So the equation that relates x and y is

$$y = \frac{20}{x}$$

NOW we find x when y is 4:

$$y = \frac{20}{x}$$

$$4 = \frac{20}{x}$$

$$(x)4 = (x)\frac{20}{x}$$

$$\frac{4x}{4} = \frac{20}{4}$$

$$x = 5$$

VARIATION PRACTICE PROBLEMS

1. y varies directly as x . When x is 7, y is 21. Find y when x is 5.

2. y varies inversely as x . When x is 2, y is 14. Find y when x is 28.