

FRACTIONS - Addition & Subtraction



↳ example 1:

$$a) \frac{3}{2} + \frac{2}{2} =$$

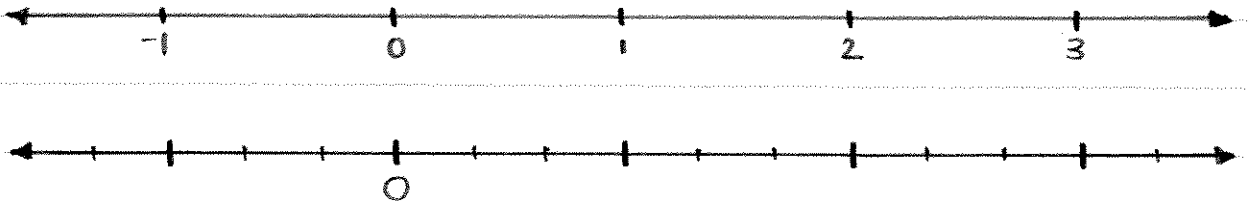
Note: Since both terms are "LIKE TERMS" we can add them.

$$b) \frac{3}{2} - \frac{5}{2}$$

$$c) \frac{5}{2} - \frac{1}{2}$$

Note: $\frac{2}{3}$ ← this number is known as the NUMERATOR
⇒ how many thirds you have
↑ this number is known as the DENOMINATOR
⇒ identifies the type of fraction
{in this case, thirds}

We can divide our number line into thirds:



↳ example 2:

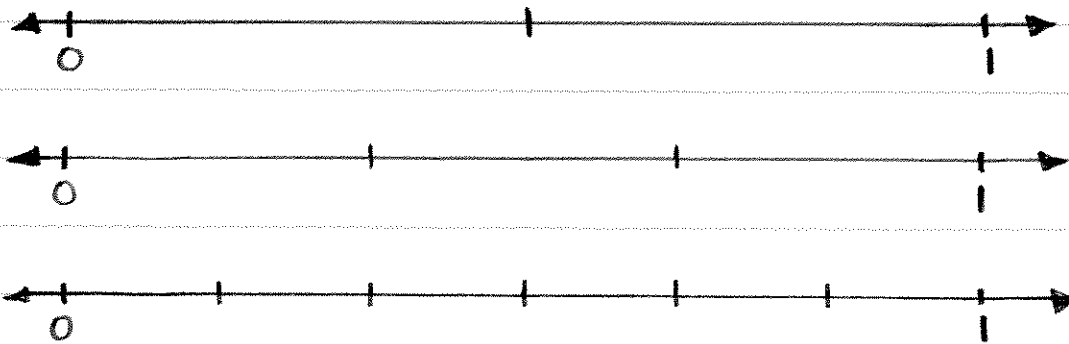
$$\begin{aligned} \text{a) } & 5 - 7 + 3 \\ & = \underline{\quad} + 3 \\ & = \underline{\quad} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{5}{3} - \frac{7}{3} + \frac{3}{3} \\ & = \\ & = \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{2}{3} + \frac{5}{3} - \frac{4}{3} \\ & = \\ & = \end{aligned}$$

what about $\frac{1}{2} + \frac{1}{3}$?

These are not like terms so we cannot add them until we make them into like terms.



$$\frac{1}{2} = \frac{\quad}{6} \quad \text{and} \quad \frac{1}{3} = \frac{\quad}{6}$$

therefore,

$$\begin{aligned} \frac{1}{2} + \frac{1}{3} \\ &= \frac{\quad}{6} + \frac{\quad}{6} \\ &= \frac{\quad}{6} \end{aligned}$$

When adding or subtracting fractions, we must find the _____ or LCD for short.

The LCD of two fractions is the smallest positive number that is divisible by BOTH denominators.

How do we compute $\frac{1}{2} + \frac{1}{3}$ without a number line?

We must first identify the LCD.
LCD = _____

Recall: multiplying any number by 1 does not change the number.

$$\frac{1}{2} \left(\quad \right) = \frac{\quad}{6}$$

$$\frac{1}{3} \left(\quad \right) = \frac{\quad}{6}$$

So $\frac{1}{2} + \frac{1}{3} = \frac{\quad}{6} + \frac{\quad}{6} = \frac{\quad}{6}$

↳ example 3:

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

The LCD is _____ since 2, 3 AND 4
all go into it evenly.

We must rewrite our fractions to
make them LIKE TERMS

$$\frac{1}{2}(\quad) + \frac{1}{3}(\quad) + \frac{1}{4}(\quad)$$

$$= \frac{\quad}{12} + \frac{\quad}{12} + \frac{\quad}{12}$$

=

↳ example 4:

$$\frac{5}{6} - \frac{2}{3} + \frac{7}{4}$$

LCD = _____

↳ example 5:

Reduce each fraction to lowest terms

a) $\frac{6}{8}$

b) $\frac{12}{9}$

c) $\frac{100}{28}$

↳ example 6:

a) write $\frac{2}{3}$ as an equivalent

fraction with a denominator of 6.

b) write $\frac{1}{5}$ as an equivalent

fraction with a denominator of 10x

FRACTIONS - Addition & Subtraction Practice Problems

1. $\frac{1}{2} + \frac{5}{2}$

2. $\frac{1}{2} - \frac{5}{2}$

3. $\frac{4}{3} - \frac{2}{3} + \frac{5}{3}$

4. $\frac{1}{3} + \frac{1}{5}$

5. Reduce each fraction

a) $\frac{4}{8} =$

b) $\frac{15}{3} =$

c) $\frac{21}{7} =$

REDUCING FRACTIONS

↪ example 1:

Reduce the following fractions to lowest terms:

a) $\frac{6}{8}$

we ask, what is the largest number that divides evenly into the top (6) AND the bottom (8)?

_____ divides evenly into both.

$$\frac{6 \cancel{2}}{8 \cancel{2}} =$$

b) $\frac{42}{28}$

with large numbers, sometimes it's easier to divide top and bottom by small values to start.

$$\frac{42 \cancel{2}}{28 \cancel{2}} =$$

=

=

$$\frac{3}{3} = 1$$

But why?

We can divide top and bottom by 3.

$$\frac{3/3}{3/3} = \frac{1}{1} = 1$$

↳ example 2:

a) Reduce $\frac{3a}{3b}$

b) Reduce $\frac{4xy}{4y}$

c) Reduce $\frac{4xy}{4xyz}$

↳ example 3:

a) Reduce $\frac{3x^2y^3}{xy}$

$$\frac{3x^2y^3}{xy} = \frac{3 \cdot x \cdot x \cdot y \cdot y \cdot y}{x \cdot y}$$

$$= 3 \cdot \frac{x}{x} \cdot x \cdot \frac{y}{y} \cdot y \cdot y$$

$$= 3 \cdot 1 \cdot x \cdot 1 \cdot y \cdot y$$

$$= 3 \cdot x \cdot y \cdot y$$

$$= 3xy^2$$

ANOTHER APPROACH:

$$\frac{3x^2y^3}{xy} = \frac{3 \cdot x \cdot x \cdot y \cdot y \cdot y}{x \cdot y} = \frac{3 \cdot x \cdot y \cdot y}{1} = 3xy^2$$

b) Reduce $\frac{4x^2y^3z^5}{xy^4z^7}$

↳ example 4: Reduce:

$$a) \frac{4x}{8x}$$

$$b) \frac{42a^2}{30a}$$

$$c) \frac{102a^3b^2c^4}{114a^2b^3c^2}$$

$$d) \frac{6}{51}$$

$$e) \frac{6}{52}$$

$$f) \frac{6}{54}$$

REDUCING FRACTIONS Practice Problems

Reduce the following fractions to lowest terms:

1. $\frac{5}{10}$

2. $\frac{80}{24}$

3. $\frac{5a}{5ab}$

4. $\frac{7x^3y^2z}{x^2yz^3}$

5. a) $\frac{12}{48}$

b) $\frac{12}{50}$

c) $\frac{12}{52}$

MULTIPLYING FRACTIONS

↳ EXAMPLE 1:

$$a) \frac{1}{2} \cdot \frac{8}{3}$$

consider the question,
What is one half of 8 apples?
_____ apples.

So, what is one half of 8 thirds?
_____ thirds

Therefore: $\frac{1}{2} \cdot \frac{8}{3} = \frac{4}{3}$

Another approach:

$$\frac{1}{2} \cdot \frac{8}{3} = \frac{1 \cdot 8}{2 \cdot 3} = \frac{8}{6} = \frac{4}{3}$$

$$b) \frac{40}{3} \cdot \frac{9}{10}$$

↪ example 2:

$$a) \frac{2}{3} \cdot \frac{4}{5} =$$

$$b) \frac{1}{4} \cdot 9 =$$

$$c) \frac{2}{3} \cdot 7 =$$

$$d) \frac{2}{5} \cdot \frac{3}{5} \cdot \frac{4}{5} =$$

$$e) \left(-\frac{2}{3}\right) \left(\frac{7}{6}\right) =$$

↳ example 3:

$$a) \frac{x}{y} \cdot \frac{y}{x} =$$

$$b) \frac{ab}{c} \cdot \frac{c}{a} \cdot \frac{c}{b} =$$

$$c) \frac{72}{35} \cdot \frac{55}{108} \cdot \frac{14}{110} =$$

↳ example 4:

$$a) \left(\frac{1}{3}\right)^2 =$$

$$b) \left(-\frac{3}{4}\right)^2 =$$

$$c) \left(\frac{1}{2}\right)^2 \cdot 8 + \left(\frac{2}{3}\right)^2 \cdot 9$$

$$d) \left(-\frac{1}{2}\right)^2 - 9\left(\frac{1}{3}\right)^2$$

MULTIPLYING FRACTIONS Practice Problems

1. $\frac{1}{2} \cdot \frac{10}{7}$

2. $6\left(\frac{2}{3}\right)$

3. $\frac{2}{7} \cdot \frac{3}{7} \cdot \frac{1}{2}$

4. $\left(-\frac{1}{4}\right)\left(\frac{8}{9}\right)$

5. $\frac{21}{64} \cdot \frac{16}{5} \cdot \frac{25}{7}$

6. $\left(\frac{1}{2}\right)^3 \cdot 4$

DIVISION WITH FRACTIONS

↳ example 1:

$$\frac{8}{\frac{1}{2}} = 8 \div \frac{1}{2}$$

consider the number line:



$8 \div \frac{1}{2}$ asks "How many halves in 8 wholes?"

$8 \div \frac{1}{2} = 16$, since there are 16 halves in 8 wholes.

ANOTHER APPROACH:

$$8 \div \frac{1}{2} = 8 \cdot \frac{2}{1} = \frac{16}{1} = 16$$

↳ example 2:

a) $\frac{3}{4} \div 2$

b) $(-4) \div \frac{8}{5}$

↳ example 3:

$$a) \frac{40}{69} \div \frac{25}{46}$$

$$b) \frac{xy^2}{z} \div \frac{y}{z}$$

↳ example 4:

$$a) 12 \div \left(\frac{1}{2}\right)^2$$

$$b) \frac{3}{5} \div \frac{1}{10} + 8$$

↳ example 5:

a) what is the quotient of
 $\frac{2}{9}$ and $\frac{4}{9}$?

b) If the quotient of $\frac{4}{5}$ and $\frac{1}{10}$
is increased by 5,
what value results?

DIVISION WITH FRACTIONS practice problems

1. $4 \div \frac{1}{2}$

2. $\frac{10}{\frac{1}{5}}$

3. $\frac{16}{27} \div \frac{20}{36}$

4. $\frac{a^2bc^3}{12} \div \frac{abc}{3}$

5. $\left(\frac{1}{3}\right)^2 \div \frac{5}{9}$

6. what is the quotient of $\frac{8}{9}$ and $\frac{2}{3}$?

MORE FRACTIONS - Addition & Subtraction

↳ example 1:

$$a) \quad \frac{2}{3} + \frac{1}{4} - \frac{5}{6}$$

LCD =

$$\frac{2}{3} (\quad) + \frac{1}{4} (\quad) - \frac{5}{6} (\quad)$$

=

=

=

$$b) \quad \frac{x}{2} + \frac{1}{4}$$

LCD =

$$\frac{x}{2} (\quad) + \frac{1}{4} =$$

=

NOTE: We didn't need to change $\frac{1}{4}$ in example 1 b) since its denominator is the LCD

↳ example 2:

$$a) \frac{5}{9} - \left(-\frac{1}{6}\right)$$

$$b) \frac{3}{x} + \frac{2}{5}$$

LCD =

$$\frac{3}{x} \left(\quad\right) + \frac{2}{5} \left(\quad\right)$$

=

=

MORE FRACTIONS

Practice Problems

1. $\frac{1}{4} + \frac{2}{5} - \frac{3}{2}$

2. $\frac{x}{4} + \frac{2}{5}$

3. $-\frac{1}{4} - \left(-\frac{1}{2}\right)$

4. $\frac{2}{x} - \frac{1}{3}$

KUNG - FU FRACTIONS

consider $4 \cdot \frac{1}{2}$

$$4 \cdot \frac{1}{2} = \frac{4}{1} \cdot \frac{1}{2} = \frac{4 \cdot 1}{1 \cdot 2} = \frac{4}{2} = \frac{2}{1} = \boxed{2}$$

can we calculate this mentally?

$4 \cdot \frac{1}{2}$ We say to ourselves,
2 goes into 4 TWO
times and TWO times
1 is $\boxed{2}$.

Now try these:

a) $8\left(\frac{5}{2}\right)$ — goes into —
— times and
— times — is —

$$\text{so } 8\left(\frac{5}{2}\right) =$$

$$\text{b) } 10\left(\frac{5}{2}\right) =$$

$$\text{c) } 9\left(\frac{4}{3}\right) =$$

$$d) 14 \left(\frac{5}{7} \right) =$$

Unfortunately, this method doesn't always work.

Consider $7 \left(\frac{3}{2} \right)$

2 does not go into 7 evenly, so we must use another method.

$$7 \left(\frac{3}{2} \right) = \frac{7}{1} \cdot \frac{3}{2} = \frac{7 \cdot 3}{1 \cdot 2} = \boxed{\frac{21}{2}}$$

Now consider

$$8 \left(\frac{3}{4} + \frac{3}{2} \right)$$

By the order of operations, we must evaluate the parenthesis first.

But, by Kung-Fu, we can take advantage of the distributive property to make the problem easier

$$8\left(\frac{3}{4} + \frac{3}{2}\right)$$

$$= 8\left(\frac{3}{4}\right) + 8\left(\frac{3}{2}\right)$$

=

=

NOW try these:

a) $6\left(\frac{2}{3} - \frac{5}{6}\right)$

b) $12\left(\frac{5}{6} + \frac{3}{4}\right)$

KUNG-FU FRACTIONS practice problems

1. Evaluate mentally, if possible:

a) $10 \cdot \frac{3}{2}$

b) $12 \left(\frac{5}{6} \right)$

c) $5 \left(\frac{1}{2} \right)$

2. Evaluate:

a) $14 \left(\frac{3}{2} + \frac{1}{7} \right)$

b) $9 \left(\frac{4}{3} + \frac{1}{9} \right)$

COMPLEX FRACTIONS

A complex fraction is _____

↳ example 1:

$$\frac{\frac{3}{4}}{\frac{5}{6}}$$

we can multiply the top
and bottom by the LCD
LCD = _____

$$\frac{\frac{3}{4} ()}{\frac{5}{6} ()} =$$

Another method:

$$\frac{\frac{3}{4}}{\frac{5}{6}} = \frac{3}{4} \div \frac{5}{6} =$$

=

But this latter method ONLY works
when you have _____

↳

EXAMPLE 2:

$$\frac{2}{3} + \frac{3}{4}$$

$$\frac{5}{6} - \frac{3}{2}$$

LCD = _____

$$\frac{\frac{2}{3} + \frac{3}{4}}{\frac{5}{6} - \frac{3}{2}} = \frac{12\left(\frac{2}{3}\right) + 12\left(\frac{3}{4}\right)}{12\left(\frac{5}{6}\right) - 12\left(\frac{3}{2}\right)}$$

$$= \frac{4(2) + 3(3)}{2(5) - 6(3)}$$

$$= \frac{8 + 9}{10 - 18}$$

$$= \frac{17}{-8}$$

$$= -\frac{17}{8}$$

Note: Remember, $\frac{17}{-8} = \frac{-17}{8} = -\frac{17}{8}$

We prefer to write $-\frac{17}{8}$ as
our final answer.

↪

example 3:

a)

$$\frac{\frac{1}{2} - \frac{1}{3}}{\frac{1}{2} + \frac{1}{3}}$$

LCD =

b)

$$\frac{1 + \frac{3}{4}}{1 - \frac{3}{4}}$$

LCD =

↳ example 4:

$$2 - \frac{2}{3} + \frac{3}{4}$$

$$\frac{1}{2} - 3 + \frac{5}{6}$$

COMPLEX FRACTIONS Practice Problems

Evaluate:

1.

$$\frac{\frac{2}{3}}{\frac{1}{4}}$$

2.

$$\frac{\frac{1}{2} + \frac{3}{4}}{\frac{4}{3} + \frac{1}{6}}$$

3.

$$\frac{\frac{1}{3} + 2 - \frac{1}{6}}{\frac{2}{9} + \frac{5}{6} - 1}$$