

National Standard for Calculus Grades

Study Results

The Typical Student Definitions

Panel members discussed characteristics of the typical student at an A-level, B-level, C-level, and D-level by participating in a threaded discussion. The next four sections represent the final summary of their discussion. Panel members agreed to the written summary before making their judgments on question difficulty.

The Typical A-Level Student

- They are comfortable with algorithmic skills and procedures.
- They understand proper use of mathematical notation.
- They are able to apply calculus in application settings and new settings that they have not seen before.
- They understand the meaning of concepts and the connections between concepts.
- They are proficient with algebraic and trigonometric skills, and they are able to extend those skills to calculus.
- They have good problem-solving skills. A-level students are able to solve a problem multiple ways and develop efficient methods for problem-solving.
- They understand important definitions and theorems and are able to use them (e.g., Intermediate Value Theorem, Mean Value Theorem, Fundamental Theorem of Calculus).
- They are able to solve multi-step problems by synthesizing concepts.
- They are able to solve nonroutine problems by applying their understanding in novel or slightly different contexts and making connections.
- They are able to interpret the results of a problem and check for reasonableness of answers.
- They understand both theory and applications of calculus.
- They know and use differentiation and antidifferentiation rules correctly.
- They are able to translate words into mathematics and communicate mathematics.
- They understand subtleties, such as where a function is differentiable or how to resolve a question of accumulation when a rate of change is given in an applied setting.
- They are comfortable with a “multiple representations of functions” approach. A-level students understand the connections between representations and have the ability to move from one representation to another.
- They are able to use mathematical arguments to justify work or answers.
- They are able to chain together a reasonably logical progression of steps to produce valid mathematical arguments.

The Typical B-Level Student

- They tend to be quite proficient in algorithmic calculus skills, occasionally faulting in mathematical notation. Tasks that are straightforward and formulaic are usually completed with little error along the way.
- They are quite effective in executing predictable tasks, and they are willing to venture beyond if the way looks somewhat familiar. They generally prefer the "diligent" and "eventual" solution path rather than the "insightful" one.
- They may be good at solving application problems that they've seen before but have trouble adapting the material to truly novel settings.
- They tend to have difficulty with multi-step problems, especially non-standard problems that are unlike those they have seen in class or on homework. However, they can understand the solution to a complicated problem, even if they could not solve it without help.
- They have less well-developed algebraic facility than the A students. Their precalculus background can be remembered when reminders are given for the less obvious concepts and skills. But they sometimes can run into trouble when simplifying more complicated algebraic expressions.
- They know the rules for derivatives and integrals and can apply them in standard settings.
- They know the statements of the standard calculus theorems.
- They can usually tell how a function behaves given its derivative.
- They tend to stumble when there is conceptual knowledge combined with language (e.g., problems with language about rates of change, average rates of change, average values, etc.).
- They can be confused by variations in problems which require more conceptual understanding.
- They sometimes do not attempt to justify the reasonableness of an answer and often will not search for an alternate approach to a problem that might yield additional insight.
- They tend to take a more mechanical approach to solving problems than A-level students. They also take longer to do the problems and make more simple mistakes. On the other hand, they have good calculus skills, moderate conceptual understanding, and can apply these skills and understanding in different settings.
- They might not have the fluency with the material that A students do. As a result, they may not be able to make all of the interpretations that need to be made through the different representations of information which is presented.

The Typical C-Level Student

- They know basic differentiation and antidifferentiation rules and the main applications of calculus (e.g., equation of tangent line, extrema of a function, finding area), but C-level students have some difficulty with algebra and miss connections between concepts.

- They understand the mechanics of algorithms, but C-level students have difficulty with notation and make computational errors.
- They are not able to apply mathematics to unfamiliar settings due to their weaker conceptual understanding.
- They are able to apply algorithms correctly, but they often have difficulty with problems involving a mathematical model due to an inability to find a mathematical formulation for the situation to be modeled.
- They seek a series of steps that can be mechanically applied to solve problems.
- They are able to begin a “standard” problem but have difficulty with pattern recognition and often may not be able to complete all steps of a problem.
- They have difficulty in translating words into mathematics and communicating mathematics.
- They are able to apply a strategy that has been seen before in solving a problem.
- They are able to differentiate polynomials, use the product and quotient rules most of the time, and antidifferentiate common functions, but C-level students have difficulty with using the chain rule.
- They often find the following topics to be challenging: continuity, limit definition of the derivative, implicit differentiation, applications of rates of change, related rates, inverse trigonometric functions, and application of the Mean Value Theorem.
- They are able to solve routine problems.
- They often do not check their work or answers for reasonableness.
- They have fairly good procedural knowledge and skills, but C-level students are often weak in algebraic and trigonometric skills and problem-solving ability.
- They are limited in their conceptual understanding.
- They are not able to apply knowledge or concepts to new or unfamiliar settings.

The Typical D-Level Student

- They have difficulty carrying out even the most routine of algorithmic processes.
- They usually are not capable of applying mathematical concepts they have learned because their conceptual understanding is very weak.
- They struggle to use appropriate notation.
- Their modeling skills for geometric and physical problems are poor, and they are very unlikely to solve even the simplest of those problems.
- They struggle to complete most problems completely correctly and earn most of their points on exams through partial credit.
- They have limited knowledge of calculus techniques, such as applying differentiation/integration rules. The same is true about their level of conceptual understanding. For example, from the fact that a function is increasing, they can conclude that the derivative is positive.
- They have limited algebraic skills, and even less conceptual ability. They will often make algebraic mistakes that either turn the problem into something so hard that it is

impossible to carry out additional steps, or turn it into something that becomes completely trivial.

- They often can mime basic procedures at a level that permits them to pass the course. Often, much of their work is actually unrelated to the solution, but shows a limited recollection (as opposed to understanding) of some procedural techniques.