Chapter 9 Overview

<u>Section 9.1: Sequences</u>

- Be able to determine if a given sequence converges or diverges.
- Know that sequences can be represented by $f(n)$ or a_n, where *n* is a set of integers.
- Be able to find a formula for the *n*th term of a sequence.
- For some sequences, in order to examine $\lim_{n\to\infty} a_n = \lim_{n\to\infty} f(n)$, we look at $\lim_{x\to\infty} f(x)$, where *x* is a real number. This allows us to use L'Hopital's rule for differentiable functions.
- Be able to evaluate limits of sequences that oscillate in sign, i.e. $\lim_{n\to\infty} (-1)^n a_n$:

  i. If $\lim_{n\to\infty} a_n = 0$, then $\lim_{n\to\infty} (-1)^n a_n = 0$.

  ii. If $\lim_{n\to\infty} a_n \neq 0$, then $\lim_{n\to\infty} (-1)^n a_n$ does not exist.

<u>Section 9.2: Monotone Sequences</u>

- Be able to determine if a sequence is increasing, decreasing, strictly increasing, or strictly increasing (or if any of these properties hold eventually).
- The monotonicity of a sequence can be examined in one of three ways: by looking at the difference $a_{n+1} - a_n$, the ratio $\frac{a_{n+1}}{a_n}$, or the derivative for a differentiable function.

<u>Section 9.3: Infinite Series</u>

- By definition, an infinite series $\sum_{k=1}^{\infty} u_k$ converges to a sum *S* only if the sequence of its partial sums $\{S_n\}_{n=1}^{\infty}$ converges to *S*. $\left(\lim_{n\to\infty} S_n = S\right)$
- Be able to identify a geometric series, its first term *a* and its ratio *r*.
- A geometric series converges if $|r| < 1$ and diverges if $|r| \geq 1$; if the series converges its sum is $S = \frac{a}{1-r}$.
- Be able to find a closed form for the *n*th partial sum of a geometric series or a telescoping sum.

Section 9.4: Convergence Tests

- Divergence Test: If $\lim_{k\to\infty} u_k \neq 0$, then the series $\sum u_k$ diverges. If $\lim_{k\to\infty} u_k = 0$, we cannot conclude anything from the Divergence Test, the series may converge or diverge. The Divergence Test is a good first step when testing for the convergence or divergence of an infinite series.
- Know that the Harmonic Series $\sum_{k=1}^{\infty} \frac{1}{k}$ diverges.
- Know that a p-series $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges if $p > 1$ and diverges if $p \leq 1$.
- Integral Test: Remember this test can only be applied to a function that is both decreasing and continuous over the given interval. The test involves evaluating an improper integral, so make sure you can evaluate $\int_a^{\infty} f(x)\,dx$ easily.

Section 9.5: The Comparison, Ratio and Root Tests for Convergence

- Remember that the Comparison, Limit Comparison, Ratio and Root Tests are for series with non-negative terms only.
- To use the Comparison Test, find another series that you already know converges or diverges and determine if its terms are greater or less than the original series. You can often do this with p-series.
- The Limit Comparison Test (LCT) works well with rational expressions that involve polynomials and/or radicals.
- The Ratio Test works well with exponential expressions and factorials.
- The Root Test works well if the kth root of the general term is a simpler expression than the general term itself.
- Keep in mind that the LCT, Ratio and Root Tests involve evaluating a limit while the Comparison Test does not.

Section 9.6: The Alternating Series Test and Conditional Convergence

- The AST is for series that involve powers of – 1, i.e. $\sum (-1)^k a_k$ or $\sum (-1)^{k+1} a_k$. In order to show the sequence of a_k's is decreasing, use techniques from Section 9.2.
- Know how to bound error on the sum of a series that satisfies the hypotheses of AST, using $|S - S_n| \leq a_{n+1}$. Be able to find n for a required level of accuracy.
- Know that if $\sum |a_k|$ converges, then $\sum a_k$ converges. We say $\sum a_k$ converges absolutely.
- Know what it means for $\sum a_k$ to converge conditionally.

- Know the Ratio Test for Absolute Convergence (RTAC), which involves evaluating a limit. RTAC is most often the best first step when testing a series for absolute convergence, conditional convergence or divergence.

Section 9.7: Maclaurin and Taylor Polynomials

- Be able to find Maclaurin or Taylor polynomials for a given $f(x)$ and be able to write the polynomial in sigma notation. Remember that a Maclaurin or Taylor polynomial has a finite number of terms.
- Use a Maclaurin or Taylor polynomial to approximate function values.

Section 9.8: Maclaurin, Taylor and Power Series

- For functions that are infinitely differentiable, we define the Taylor and Maclaurin series, which are infinite series.
- Know that a power series centered at x_0 is $\sum_{k=0}^{\infty} c_k (x - x_0)^k$.
- Be able to determine the radius (*R*) and interval of convergence (*I.O.C.*) for any power series.

Section 9.10: Differentiating and Integrating Power Series

- Be able to integrate or differentiate a power series term-by-term; this can help us to find a power series representation for other functions.
- Be able to find power series by substitution into familiar series such as:

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \qquad (-\infty < x < \infty)$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k \qquad (-1 < x < 1)$$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} \qquad (-\infty < x < \infty)$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} \qquad (-\infty < x < \infty)$$