Math 3c, Exam 1 Fall '06 Karla Westphal

1. (12.1, 12.5, 12.6) Describe geometrically the shape of each graph in 3-space. One-word answers (plane, sphere, etc.) will suffice.

   (a) 3x-4y+2z= 6

   (b) $\mathbf{r}(t) = \langle 1, 1, 1 \rangle + t \langle 0, -4, 2 \rangle$

   (c) x=2

2. (1.7) Sketch the graph a parametric curve in $\mathbb{R}^2$such that all of the following hold. Label the points that correspond to t = -5 and $t = -3$ and remember to include the orientation.

   $x'(t) < 0 \quad if\ t < -3$
   $x'(t) = 0 \quad if\ t = -3$
   $x'(t) > 0 \quad if\ t > -3$
   $y'(t) < 0 \quad if\ t < -5$
   $y'(t) = 0 \quad if\ t = -5$
   $y'(t) > 0 \quad if\ t > -5$

3. (11.2) Consider the parametric curve: $x(t) = t^2 - 2$
   $y(t) = \sin t$

   (a) Find a formula for $\frac{dy}{dx}$.

   (b) Find a formula for $\frac{d^2y}{dx^2}$.

4. (12.1) Consider the point P(-1, -4, 6).

   (a) What is the distance from this point to the xz-plane?

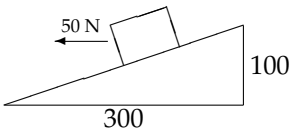
   (b) What is the distance from this point to the x-axis?

5. (12.1) Sketch in 3-space:

$$(x-1)^2 + (y+3)^2 + z^2 = 16$$

   You may supplement your picture with words if necessary to clarify what you are trying to draw.

6. (12.3) As shown below, a force of 50 Newtons acts on an object lying on an inclined plane. Find the vector components of force acting parallel and perpendicular to the plane.



7. (12.3) Let $\mathbf{u} = \langle u_1, \quad u_2 \rangle$, $\mathbf{v} = \langle v_1, \quad v_2 \rangle$, and $\mathbf{w} = \langle w_1, \quad w_2 \rangle$ be three vectors in $\mathbb{R}^2$. Prove:

$$\mathbf{u} \bullet (\mathbf{v} + \mathbf{w}) = \mathbf{u} \bullet \mathbf{v} + \mathbf{u} \bullet \mathbf{w}$$

8. (12.4) Find the volume of the parallelpiped with adjacent sides: $\mathbf{u} = \langle 2, \quad -3, \quad 0 \rangle$
$\mathbf{v} = \langle 1, \quad 0, \quad 4 \rangle$
$\mathbf{w} = \langle 2, \quad -1, \quad 2 \rangle$

9. (12.5) Find the vector equation for the line segment from P(2, -3, 5) to Q(-3, -4, 8).

10. (12.6) Find the equation of the plane through the points P(4,1,1), Q(-1, 0, 2), and R(3, -3, 2)