

Math 3c, Exam 1

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Directions: Show all work and fully justify all answers. Simplify to the extent we have in class. If you have questions about whether an answer needs further justification or simplification, ASK!

1. (1.7) Sketch the curve below by eliminating the parameter. Be sure to include the orientation, if there is one.

$$\begin{aligned}x &= t^2 \\ y &= \sqrt{t}\end{aligned}$$

2. (11.2) Consider the parametric curve below:

$$\begin{aligned}x &= \tan t \\ y &= t^2 + t\end{aligned}$$

- (a) Find a formula for $\frac{dy}{dx}$.
- (b) Find a formula for $\frac{d^2y}{dx^2}$.
3. (12.2) A force of magnitude 10 and a second force of magnitude 5 act on the same object. The angle between the two forces is 60° . Find the magnitude of the resultant force.
4. (12.5, 13.1) Give the vector equation of the line through $P(2, -4, 3)$ parallel to the line whose parametric equations are:

$$\begin{aligned}x &= 4 - t \\ y &= 3t \\ z &= 5 + 2t\end{aligned}$$

5. (12.4, 11.2, 12.2)

- (a) If \mathbf{u} , \mathbf{v} and \mathbf{w} are three non-zero vectors in 3-space, state the geometric significance of the following equation:

$$\begin{vmatrix} \leftarrow & \mathbf{u} & \rightarrow \\ \leftarrow & \mathbf{v} & \rightarrow \\ \leftarrow & \mathbf{w} & \rightarrow \end{vmatrix} = 0$$

- (b) If $x = x(t)$ is a parametric curve in 2-space, what conditions guarantee that the graph will have a horizontal tangent when $t = t_0$?
- (c) Explain what it means for two non-zero vectors, \mathbf{u} and \mathbf{v} , to be parallel. [You can give an equation or a description in words.]
6. (12.1) Sketch $z = \sqrt{y}$ in 3-space.
7. (12.3) Calculate the work done by the force $\mathbf{F} = \langle 2, -3, 5 \rangle$ acting on an object as it moves from $P(2,0,-4)$ to $Q(3, 1,2)$. Suppose that force is measured in Newtons and distance in meters so that work is given in Newton-meters (Nm).
8. (12.5) Determine whether the two lines described below are parallel, intersecting, or skew. If the lines intersect, find the point of intersection.
- | | |
|---|---------------|
| <u>Line 1</u> | <u>Line 2</u> |
| $\mathbf{r}(t) = \langle 3, 2, -1 \rangle + t \langle 2, 2, -1 \rangle$ | $x = 3 - t$ |
| | $y = 4 + t$ |
| | $z = 3 + 2t$ |
9. (12.6) Find the distance between the two parallel planes given below:
- | | |
|-----------------|----------------|
| <u>Plane 1</u> | <u>Plane 2</u> |
| $2x+4y-3z +8=0$ | $4x+8y-6z=4$ |
10. (12.6) Find the equation for the plane containing points $P(1,0,-2)$, $Q(2,2,2)$, and $R(0,1,3)$.