Math 3c, Exam 2

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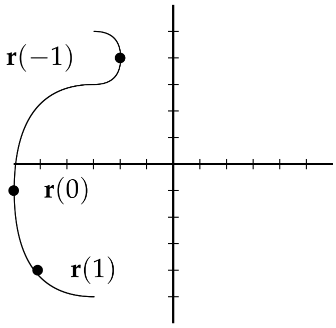
1. (12.7) Sketch
$$-x^2 + y^2 + \frac{z^2}{4} = 1$$
by first sketching the trace in each of the three coordinate planes and then sketching the surface in 3-space. On the two-dimensional graphs be sure that the location of any vertices is clear and that any asymptotes are drawn in.

2. (12.8) Convert the following equation to rectangular coordinates and then sketch the graph in 3-space.
$$r = 6\sin\theta$$

3. (13.2) Let $\mathbf{r}(t) = \left\langle 2\sin t - 3, \quad 4\cos t + 1 \right\rangle$
   (a) Sketch $\mathbf{r}(t)$, including the orientation.
   (b) Calculate $\mathbf{r}\left(\frac{2\pi}{3}\right)$. Sketch this vector in standard position.
   (c) Calculate $\mathbf{r}'\left(\frac{2\pi}{3}\right)$. Sketch this vector in translated position, with its tail at the tip of $\mathbf{r}\left(\frac{2\pi}{3}\right)$.
   (d) Sketch the tangent line at $t = \frac{2\pi}{3}$.
   (e) Find an equation for this tangent line in vector form.
   (f) Find an equation for this line by expressing y as a function of x.

4. (13.2) Let f(t) be a differentiable real-valued function and $\mathbf{r}(t) = \left\langle x(t), \quad y(t) \right\rangle$ be a differentiable vector-valued function. Prove:
$$\frac{d}{dt}\mathbf{r}\left(f(t)\right) = f'(t)\mathbf{r}'\left(f(t)\right)$$

5. (13.3-13.5) The following graph shows the graph of a smooth vector-valued function $\mathbf{r}(t)$, with three points labelled. Refer to the graph as you answer the questions that follow. (Each tick mark represents a length of one unit.)



(a) Rank in order from smallest to greatest: $\kappa(-1), \kappa(0), \kappa(1)$.

(b) Sketch the osculating circles at t = -1, 0, and 1.

(c) Rank in order from smallest to greatest: $\int_{-1}^{0} ||\mathbf{r}'(t)||\ dt$, $\int_{0}^{1} ||\mathbf{r}'(t)||\ dt$, $\int_{-1}^{1} ||\mathbf{r}'(t)||\ dt$

(d) Sketch $\mathbf{T}(1)$ and $\mathbf{N}(1)$ in translated position, so that their tails are at $\mathbf{r}(1)$.

(e) Put an "e" on the curve at the point where **N** does not exist.

6. (13.3) Find the arc length parametrization for $\mathbf{r}(t) = \langle \cos t, \quad \sin t, \quad t \rangle$ with t=0 as a reference point.

7. (14.3) Let $f(x, y) = \cos(x^2 y)$

   (a) Calculate $\frac{\partial f}{\partial x}$.

   (b) Calcualte $\frac{\partial^2 f}{\partial y \partial x}$.

8. (14.1, 14.4) Let $f(x, y) = \sqrt{xy}$.

   (a) Sketch the domain of this function in 2-space.

   (b) Sketch a contour plot for this function, including the level curves z = 0, z= 1, and z=2.

   (c) Calculate $\bigtriangledown f(1, 1)$.

   (d) Sketch $\bigtriangledown f(1, 1)$ on your contour plot so that its tail is at the point (1,1).

9. (14.4) Let $f(x, y) = x^2 y + x$. Find an equation for the tangent plane to this function at (1,2).