Math 3c, Exam 2

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Directions: Show all work. Simplify answers to the extent we have in class; if you are not sure whether an answer needs further simplification, ask! Record all work and answers together on separate paper unless you are adding something to a drawing on the test itself.

1. (12.7, 14.1) Let $w = f(x,y,z) = x^2 + \frac{y^2}{4} - \frac{z^2}{9}$
   (a) Sketch the level surface w = -1.
   (b) Sketch the level surface w = 0 .

   Rough sketches are ok for both level surfaces; you do not need to draw any 2-dimensional traces unless you find them helpful in sketching your final result.

2. (12.8) Sketch each surface/region described below in spherical coordinates.
   (a) $\theta = \pi$
   (b) $\phi = \frac{\pi}{2}$
   (c) $\phi \geq \frac{2\pi}{3}$, $\rho \leq 5$

3. (13.1, 13.2) Let $\mathbf{r}(t) = \langle 2\sin t, \quad \cos t \rangle$
   (a) Sketch $\mathbf{r}(t)$, including its orientation.
   (b) Calculate $\mathbf{r}\left(\frac{\pi}{6}\right)$.
   (c) Sketch $\mathbf{r}\left(\frac{\pi}{6}\right)$ in standard position.
   (d) Calculate $\mathbf{r}'\left(\frac{\pi}{6}\right)$.
   (e) Sketch $\mathbf{r}'\left(\frac{\pi}{6}\right)$ in translated position with its tail at the tip of $\mathbf{r}\left(\frac{\pi}{6}\right)$.
   (f) Sketch the tangent line at $t = \frac{\pi}{6}$.
   (g) Find a vector equation for the line in part (f).
   (h) Give an equation for the line in part (f) by expressing y as a function of x.

4. (13.3) Let $\mathbf{r}(t) = \left\langle \frac{\sqrt{6}}{3}t^3, \quad t, \quad \frac{3}{2}t^2 \right\rangle$ be a function representing an object's position, $\mathbf{r}$, as a function of time, t.

   (a) Find the object's displacement over the time interval [0,2].

   (b) Find the distance traveled by the object over the time interval [0,2]. (Set up, but don't evaluate the appropriate integral.)

5. (13.2) Let $\mathbf{r}_1(t) = \langle x_1(t), \quad y_1(t) \rangle$ and $\mathbf{r}_2(t) = \langle x_2(t), \quad y_2(t) \rangle$ be two differentiable vector-valued functions. Prove:

$$\frac{d}{dt}\left(\mathbf{r}_1(t) \bullet \mathbf{r}_2(t)\right) = \mathbf{r}_1'(t) \bullet \mathbf{r}_2(t) + \mathbf{r}_1(t) \bullet \mathbf{r}_2'(t)$$

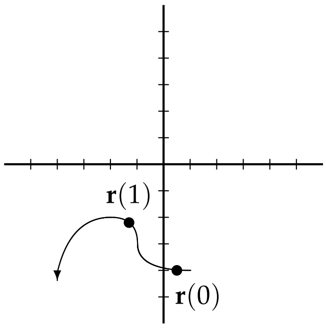
6. (13.3-13.5) The graph of a smooth curve $\mathbf{r}(t)$ is shown below with several points marked off.

   (a) Sketch **T** and **N** at point **r**(1), assuming that each tick mark on the graph represents a length of one unit.

   (b) Which is greater, $\kappa(0)$ or $\kappa(1)$?

   (c) Put a "**c**" on the graph at the point where there is no unit normal vector.

   (d) Suppose that $||\mathbf{r}'(t)||$ is constant; is it smaller than, equal to, or greater than 1?



7. (13.6) Suppose an object's acceleration is given as a function of time by the equation $\mathbf{a}(t) = \langle e^t, \quad \cos 2t \rangle$ and that the object's initial velocity (when time t=0) is $\langle 3, \quad -2 \rangle$. Find a formula for the object's velocity as a function of time.

8. (14.4) Let $f(x,y) = x^2 + y^2$.

   (a) Find an equation for L(x,y), the tangent plane to the graph of f at (1,1).

   (b) Find a formula for E(x,y) at this point.

   (c) Find a formula for $d(P, P_0)$ if P is an arbitrary point (x,y) and P_0 = (1,1).

(d) (**Extra credit: 2 points**) Prove that f is differentiable at $P_0 = (1, 1)$ by showing that $\lim_{P \to P_0} \frac{E(x,y)}{d(P,P_0)} = 0$.

9. (14.5) Let $\mathbf{r}(t) = \langle t^2, \quad \sin t \rangle$ and let $f(x, y) = x^2 y$.

   Use the chain rule to calculate $\frac{d}{dt} f(\mathbf{r}(t))$. [Note: to receive full credit you *must* use the chain rule to calculate your answer.]