Math 3c, Midterm 3

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1. (14.7) Find the tangent plane to the surface $\sin(xy) + z = -1$ at the point (0,1,-1).

2. (14.8, 14.9) Let $f(x,y) = x^3 + y^3$ and let D = $\left\{(x,y) \,\middle|\, x^2 + y^2 \leq 1\right\}$ be the solid unit disk in the xy-plane.

   (a) We know that f achieves an absolute max and min on D by the ______________theorem, which applies here since f is ______________and D is ______________and ______________. The max and min could occur at one of two types of locations: ______________or ______________.

   (b) Find the location and value of the absolute max and min. Use the method of Lagrange multipliers on the boundary.

3. (14.8)

   (a) Fill in the blank. Let z=f(x,y) be a continuous function such that all of the second partials are also continuous. If f has a relative max or min, this must occur at a critical point, which is a point where ______________= **0** (the zero vector.) To determine whether we have a max or min at such a point, we can use the second partials test. We define

   $$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

   If D = ______________, the test is inconclusive and we know NOTHING!

   (b) According to the second partials test, what must be true at a critical point for us to conclude that we have a relative max here?

   (c) According to the second partials test, what must be true at a critical point for us to conclude that we have a relative min here?

   (d) According to the second partials test, what must be true at a critical point for us to conclude that we have a saddle point here?

4. (15.5, 15.7) Let G be the solid enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the hemisphere $z = \sqrt{4 - x^2 - y^2}$. Set up, but DO NOT EVALUATE a triple integral that will allow us to calculate the volume of G:

(a) in rectangular coordinates

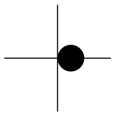
(b) in cylindrical coordinates

(c) in spherical coordinates.

5. (15.4) Consider the parametric surface defined by $\mathbf{r}(u,v) = \langle 2u, u^2, uv \rangle$. Find the surface area over the region in the uv-plane defined by $\begin{array}{l} 0 \le u \le \sqrt{3} \\ 0 \le v \le 1 \end{array}$.

6. (15.8) Find the volume of the region enclosed by the ellipsoid $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$. [Hint: Let S be the solid unit sphere in the u,v,w-coordinate system. Map S to the region enclosed by the ellipsoid and integrate over S.]

7. (15.6) Consider the circular lamina of radius 1 shown below. Suppose that its density function is $\delta(x,y) = y + 1$.



Calculate the mass of the lamina.

8. (16.2) Let $\mathbf{F}(x,y) = \langle xy, y \rangle$ be a force field. Let C be the quarter of the circle of radius 2, shown below, with counterclockwise orientation. Find the work done by $\mathbf{F}$ on a particle travelling along C.