

# Math 3c, Exam 3

Karla Westphal

Spring 2007

1. (14.7) Consider the surface defined by the equation

$$x^2 - 4y^2 + 2z = 5$$

Find the equation of the tangent plane to this surface at the point  $(1, 0, 2)$ .

2. (14.8, 14.9) Let  $f(x,y) = x^2 + y^3$  and let  $D = \{(x,y) \mid x^2 + y^2 \leq 1\}$  be the unit disk in the  $xy$ -plane.
- (a) The \_\_\_\_\_ theorem ensures that  $f$  achieves an absolute max and an absolute min on  $D$ , since  $f$  is \_\_\_\_\_ and  $D$  is \_\_\_\_\_ and \_\_\_\_\_. The max and min could occur at two types of locations: \_\_\_\_\_ and \_\_\_\_\_.
- (b) Find the location and value of the absolute max and absolute min of  $f$  on  $D$ . Use the method of Lagrange multipliers on the boundary.
3. (15.5, 15.7) Let  $G$  be the solid bounded by the surfaces  $z = x^2 + y^2$  and  $z=9$ . Write down, but don't evaluate, a triple integral that could be used to calculate the volume of  $G$  as an iterated integral:
- (a) in rectangular coordinates  
(b) in cylindrical coordinates  
(c) in spherical coordinates.
4. (15.6) A triangular lamina with vertices  $(0,0)$ ,  $(0,1)$ , and  $(1,0)$  has density function  $\delta(x,y) = y$ . Calculate the mass of the lamina.
5. (15.4) Consider the graph of the function  $z = f(x,y) = xy$  defined on the disk of radius  $\sqrt{8}$ , centered at the origin. Calculate its surface area.
6. (15.8) Let  $R$  be the parallelogram in the  $xy$ -plane whose vertices are  $(0,0)$ ,  $(1,1)$ ,  $(2,0)$ , and  $(1,-1)$ .
- (a) Find a transformation that maps a rectangle,  $S$ , in the  $uv$ -plane to  $R$  in the  $xy$ -plane.

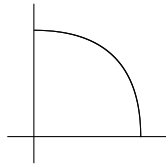
(b) Evaluate  $\iint_R (y - x) dA_{x,y}$  by using the transformation you found in part (a) to integrate over S.

7. (16.1) Let  $\mathbf{F} = \langle x, xy, xyz \rangle$

(a) Calculate  $\text{div } \mathbf{F}$ .

(b) Calculate  $\text{curl } \mathbf{F}$ .

8. (16.2) Let  $f(x,y) = xy$  and let C be the quarter circle of radius 2 shown below, oriented counterclockwise.



Evaluate  $\int_C f(x,y) ds$ .