Math 3c, Final Exam

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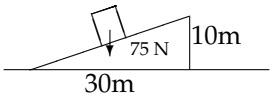
1. Let $f(x, y) = \sin(xy)$ Find the equation of the plane tangent to the graph of f at the point $\left(\pi, \frac{1}{3}\right)$.

2. As shown below, a force of 75 Newtons acts on an object lying on an inclined plane. Calculate the *vector* components of force parallel and perpendicular to the plane. [Answers must be vectors, not numbers!! Do NOT rotate the coordinate axes.]



3. Let $\mathbf{r}(t) = \langle 2\cos t, \quad 3\sin t + 5 \rangle$

   (a) Sketch $\mathbf{r}(t)$, including its orientation.

   (b) Sketch $\mathbf{r}\left(\frac{\pi}{3}\right)$ in standard position.

   (c) Calculate $\mathbf{r}'(t)$.

   (d) Calculate $\mathbf{r}'\left(\frac{\pi}{3}\right)$ and sketch this vector in translated position, with its tail at the tip of $\mathbf{r}\left(\frac{\pi}{3}\right)$.

   (e) Sketch the tangent line to the curve at the point where $t = \frac{\pi}{3}$.

   (f) Find an equation for this line in vector form.

   (g) Find an equation for this line by expressing y as a function of x.

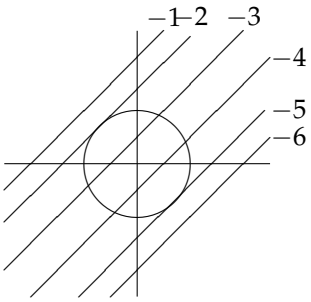
4. Let G be the portion of the sphere of radius 3 centered at the origin that lies in the first octant. Set up, but don't evaluate, a triple integral to calculate the volume of G:

   (a) in rectangular coordinates.

(b) in cylindrical coordinates.

(c) in spherical coordinates.

5. The graph below shows the unit circle together with a contour plot for the function f. Assume that the contour plot gives an accurate representation of f's behavior.



(a) What is the maximum value f achieves on the unit circle?

(b) Put a "b" on the graph at the point where this max is achieved.

(c) What is the minimum value f achieves on the unit circle?

(d) Put a "d" on the graph at the point where this max is achieved.

6. (a) Given a differentiable function f(P), what vector points in the direction of f's greatest increase?

(b) What is the curvature of a circle of radius 4?

(c) Given a smooth curve $\mathbf{r}(t); a \leq t \leq b$, what is the formula for calculating the arc length?

7. Let $\mathbf{F}(x,y) = \langle 3x^2y^2 - \sin x, \quad 2x^3y - e^y \rangle$.

(a) Verify that $\mathbf{F}$ is conservative.

(b) Find a potential function for $\mathbf{F}$.

8. Let σ be the portion of the plane $2x + 2y + z = 2$ lying above the first octant. Evaluate

$$\iint_{\sigma} x + z \, dS$$

[Hint: Solve for z as a function of x and y.]

9. (a) Let $\mathbf{F}(x,y,z)$ be a vector field whose component functions have continuous first partials. Let G be a finite solid whose boundary surface σ is oriented outward. State the Divergence Theorem.

(b) Use the Divergence Theorem to evaluate the flux of $\mathbf{F}(x,y,z) = \langle xz^2, \quad yz^2, \quad z^3 \rangle$across the outward-oriented sphere of radius 2, centered at the origin.

10. Let σ be the top half of the unit sphere, centered at the origin, with outward orientation, with boundary curve C, positively oriented. Let $\mathbf{F}(x, y, z) = \langle x + y, y + z, x + z \rangle$. Calculate $\int_C \mathbf{F} \bullet \mathbf{T}\, ds$ two ways:

    (a) By evaluating this as a line integral.

    (b) By using Stokes' theorem to evaluate an appropriate surface integral.