Math 3c, Final Exam

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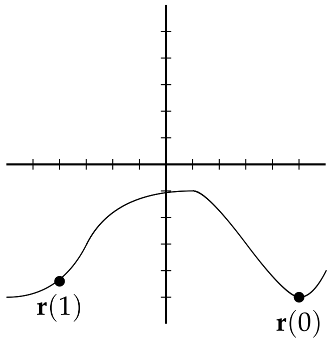
Directions: Show all work and justify all answers. Simplify to the extent we have in class; ask if you're not sure whether an answer needs further simplification. All work and answers should be recorded together on separate paper.

1. Let $\mathbf{r}(t) = \langle e^t, \quad t^2 + t \rangle$.
   (a) Find a formula for $\frac{dy}{dx}$.
   (b) Find a formula for $\frac{d^2y}{dx^2}$.
2. Calculate the distance between the parallel planes given below:
$$\begin{aligned} 2x - y + 3z &= 4 \\ -2x + y - 3z &= 5 \end{aligned}$$
3. Let $\mathbf{r}(t) = \left\langle \frac{t^3}{3}, \quad \frac{t^2}{2} \right\rangle; t \geq 0$. Find the arc length parametrization for this curve with t=0 as the reference point.

4. The graph of a smooth curve $\mathbf{r}(t)$ is drawn below, with two points (drawn as points rather than vectors) indicated. For this problem, you may record your answers directly on the test. Assume that each tick mark on the grid indicates a lenght of one unit.



   (a) Draw $\mathbf{T}(0)$ in translated position so that its tail is at the tip of $\mathbf{r}(0)$.

   (b) Draw $\mathbf{N}(0)$ in translated position so that its tail is at the tip of $\mathbf{r}(0)$.

   (c) Put a "**c**" on the graph at a point where the unit normal vector does not exist.

   (d) Draw the vector $\int_0^1 \mathbf{r}'(t)\,dt$ in translated position so that its tail is at the tip of $\mathbf{r}(0)$.

   (e) Which is bigger: $\kappa(0)$ or $\kappa(1)$?

5. Let R be the region enclosed by the triangle whose vertices are (-1,0), (1,0), and (-1,4). Let $f(x,y)$ = x-xy.

   (a) How do we know that f achieves an absolute max and an absolute min on R? [Be specific; name the theorem and tell me what conditions must be satisfied for the theorem to apply.]

   (b) Find the location and value of the absolute max and the absolute min of f on R.

6. Let R be the region enclosed by the ellipse $x^2 + \frac{y^2}{4} = 1$. Evaluate

$$\iint_R (xy+2)\,dA$$

   [Hint: Start by finding a transformation that maps the unit disk to R.]

7. Let σ be the surface of the unit sphere, centered at the origin. Evaluate:

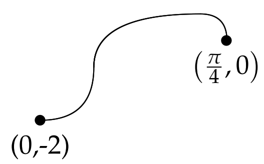
$$\iint_\sigma z^2\,dS$$

8. Let $\mathbf{F}(x, y) = \langle ye^{xy} + \sec^2 x, \quad xe^{xy} - 2 \rangle$.

   (a) Verify that $\mathbf{F}$ is conservative.

   (b) Find a potential function ϕ for $\mathbf{F}$.

   (c) Use your answer to part (b) to evaluate $\int_C \mathbf{F} \bullet \mathbf{T}\, ds$ where C is the curve shown below, traced from left to right:



9. Let G be the unit cube located in the first octant with one corner at the origin. Let σ be the boundary surface of this cube and let σ have outward orientation. Let $\mathbf{E} = \langle xy, \quad z - y, \quad x^2y^3 \rangle$. Calculate the flux of $\mathbf{E}$ across σ using any (legitimate) method.

10. Let σ be portion of the paraboloid $z = x^2 + y^2 - 9$ lying on and below the xy-plane. Let σ have downward orientation. Let C be its boundary curve, oriented positively with respect to σ. Let $\mathbf{F} = \langle x - y, \quad x^2 z, \quad y \rangle$.

   (a) Evaluate $\int_C \mathbf{F} \bullet \mathbf{T}\, ds$ as a line integral.

   (b) Use Stokes' theorem to evaluate $\int_C \mathbf{F} \bullet \mathbf{T}\, ds$ by evaluating an appropriate surface integral.