Section 12.1

(10) (a) $d = 3$ (d) $d = \sqrt{13}$

(b) $d = 2$ (e) $d = \sqrt{34}$

(c) $d = 5$ (f) $d = \sqrt{29}$

Section 12.2

(54) Thm. 12.2.6

(a) Prove $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ in $\mathbb{R}^2$.

Let $\vec{u}, \vec{v}$ be in $\mathbb{R}^2$. Then $\vec{u} = \langle u_1, u_2 \rangle$, $\vec{v} = \langle v_1, v_2 \rangle$.

$$
\begin{aligned}
\vec{u} + \vec{v} &= \langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle \\
&= \langle u_1 + v_1, u_2 + v_2 \rangle \\
&= \langle v_1 + u_1, v_2 + u_2 \rangle \\
&= \langle v_1, v_2 \rangle + \langle u_1, u_2 \rangle \\
&= \vec{v} + \vec{u}
\end{aligned}
$$

as needed.

(c) Prove $\vec{u} + \vec{0} = \vec{0} + \vec{u} = \vec{u}$ in $\mathbb{R}^2$.

Let $\vec{u}$ be in $\mathbb{R}^2$. Then $\vec{u} = \langle u_1, u_2 \rangle$.

$$
\begin{aligned}
\vec{u} + \vec{0} &= \langle u_1, u_2 \rangle + \langle 0, 0 \rangle \\
&= \langle u_1 + 0, u_2 + 0 \rangle \\
&= \langle 0 + u_1, 0 + u_2 \rangle \\
&= \langle 0, 0 \rangle + \langle u_1, u_2 \rangle \\
&= \vec{0} + \vec{u}
\end{aligned}
$$

as needed.

(54) (c) continued

Further, $\vec{u} + \vec{0} = \langle u_1 + 0,\ u_2 + 0 \rangle$

$= \langle u_1, u_2 \rangle$

$= \vec{u}$ as needed.

(e) Prove $k(\ell \vec{u}) = (k\ell)\vec{u}$ in $\mathbb{R}^2$.

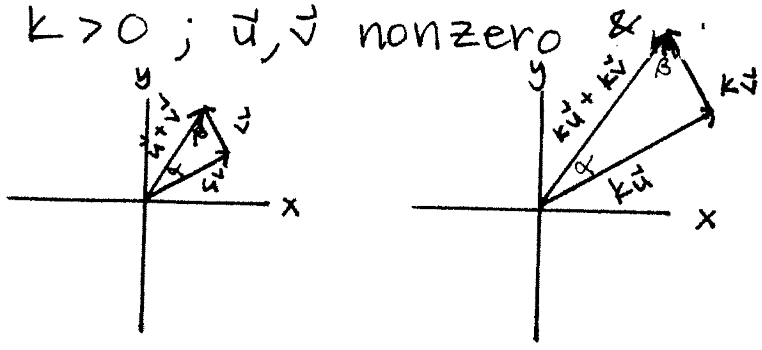
Let $\vec{u}$ be in $\mathbb{R}^2$. Then $\vec{u} = \langle u_1, u_2 \rangle$.

$$
\begin{aligned}
k(\ell \vec{u}) &= k \langle \ell u_1, \ell u_2 \rangle \\
&= \langle k\ell u_1, k\ell u_2 \rangle \\
&= k\ell \langle u_1, u_2 \rangle \\
&= (k\ell)\vec{u}
\end{aligned}
$$

as needed.

(56) Thm 12.2.6 (f)

Prove $k(\vec{u} + \vec{v}) = k\vec{u} + k\vec{v}$ geometrically. Assume $k > 0$; $\vec{u}, \vec{v}$ nonzero & $\vec{u} \nparallel \vec{v}$.

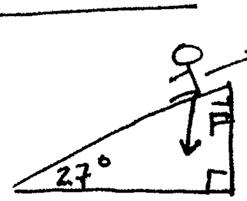


By similar triangles we have: $\dfrac{\vec{u}}{k\vec{u}} = \dfrac{\vec{v}}{k\vec{v}} = \dfrac{\vec{u} + \vec{v}}{k\vec{u} + k\vec{v}}$

so that $k\vec{u} + k\vec{v} = k(\vec{u} + \vec{v})$ as needed.

Section 12.3

(30)



$m = 34$ kg

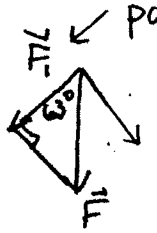
$g = 9.8\ m/s^2$

force of gravity: $f = m \cdot a$

$f = 34(9.8)$

$= 333.2$ N

So $\|\vec{F}\| = 333.2$ N



$\vec{F_1}$ ← parallel to slide

$\vec{F_2}$ ← ⊥ to slide

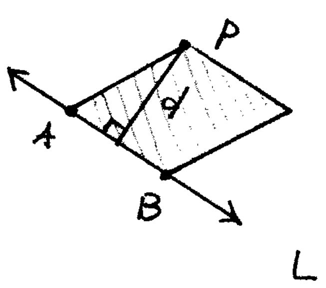
$\|\vec{F_1}\| = \|\vec{F}\| \cos 63° = \boxed{333.2 \cos 63° \text{ N}}$ force to prevent child from sliding

$\|\vec{F_2}\| = \|\vec{F}\| \sin 63° = \boxed{333.2 \sin 63° \text{ N}}$ force exerted by child on slide

Section 12.4

(26)

Consider the area of the parallelogram.



Area $= \|\vec{AP} \times \vec{AB}\|$

but also

Area $= d\,\|\vec{AB}\|$ (altitude · base)

So $\|\vec{AP} \times \vec{AB}\| = d\,\|\vec{AB}\|$

and $d = \dfrac{\|\vec{AP} \times \vec{AB}\|}{\|\vec{AB}\|}$ as needed.

Section 12.4 continued

(38) Prove parts (b) & (c) of Thm 12.4.3.

(b) Show $\vec{u} \times (\vec{v}+\vec{w}) = (\vec{u}\times\vec{v}) + (\vec{u}\times\vec{w})$.

Proof: Let $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^3$. Then $\vec{u} = \langle u_1, u_2, u_3\rangle$, $\vec{v} = \langle v_1, v_2, v_3\rangle$ & $\vec{w} = \langle w_1, w_2, w_3\rangle$.

We have: $\vec{v}+\vec{w} = \langle v_1+w_1,\ v_2+w_2,\ v_3+w_3\rangle$

so that
$$\vec{u}\times(\vec{v}+\vec{w}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1+w_1 & v_2+w_2 & v_3+w_3 \end{vmatrix}$$

$$= \langle u_2(v_3+w_3) - u_3(v_2+w_2),\ -(u_1(v_3+w_3) - u_3(v_1+w_1)),\ u_1(v_2+w_2) - u_2(v_1+w_1)\rangle \quad (*)$$

Also:
$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \langle u_2v_3 - u_3v_2,\ -(u_1v_3 - u_3v_1),\ u_1v_2 - u_2v_1\rangle$$

and
$$\vec{u}\times\vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \langle u_2w_3 - u_3w_2,\ -(u_1w_3 - u_3w_1),\ u_1w_2 - u_2w_1\rangle$$

so that $(\vec{u}\times\vec{v}) + (\vec{u}\times\vec{w})$

$$= \langle u_2(v_3+w_3) - u_3(v_2+w_2),\ -(u_1(v_3+w_3) - u_3(v_1+w_1)),$$
$$u_2(v_2+w_2) - u_2(v_1+w_1)\rangle$$

which agrees with $(*)$

So $\vec{u}\times(\vec{v}+\vec{w}) = (\vec{u}\times\vec{v}) + (\vec{u}\times\vec{w})$ as needed.

(c) Show $(\vec{u}+\vec{v})\times\vec{w} = (\vec{u}\times\vec{w}) + (\vec{v}\times\vec{w})$.

(this can be shown "brute force" as in part (b); alternatively you can use the result from (b) to prove (c) as shown here)

(38) (c) continued

Proof:

$$(\vec{u}+\vec{v})\times\vec{w} = -(\vec{w}\times(\vec{u}+\vec{v})) \quad \text{(from part (a))}$$

$$= -\vec{w}\times(\vec{u}+\vec{v}) \quad \text{(from part (d))}$$

$$= (-\vec{w}\times\vec{u}) + (-\vec{w}\times\vec{v}) \quad \text{(from part (b))}$$

$$= -(\vec{w}\times\vec{u}) - (\vec{w}\times\vec{v}) \quad \text{(from part (d))}$$

$$= (\vec{u}\times\vec{w}) + (\vec{v}\times\vec{w}) \quad \text{(from part (a))}$$

as needed.

(40) Prove part (b) of Thm.12.4.1 for 3×3 determinants. (for the first 2 rows)

Proof: Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$.

Then $\det(A) = |A| = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$

$$= a_1b_2c_3 - a_1b_3c - a_2b_1c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 \quad (*)$$

Interchanging rows 1 & 2:

$$B = \begin{bmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

and $\det(B) = |B| = b_1(a_2c_3 - a_3c_2) - b_2(a_1c_3 - a_3c_1) + b_3(a_1c_2 - a_2c_1)$

$$= a_2b_1c_3 - a_3b_1c_2 - a_1b_2c_3 + a_3b_2c_1 + a_1b_3c_2 - a_2b_3c_1$$

$$= -(a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1)$$

$$= -|A|$$

as needed.

(40) cont'd

Use part (b) of Thm. 12.4.1 to prove (a).
(for 3 x 3 determinants)

Proof : Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$.

We'll call $\det(A) = a$.

If we interchange rows 1 & 2, we have the same matrix(*). By part (b), the determinant of this matrix is $\det(B) = -a$. Since A & B are the same matrix, their determinants must be the same. That is,

(*) $B = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$

$$a = -a$$

$$\Rightarrow 2a = 0$$

$$\Rightarrow a = 0$$

So $\det(A) = 0$ as needed.