# Section 15.5

(36) Proof:

$$\iiint_G f(x)g(y)h(z)\,dV = \int_a^b \int_c^d \int_k^\ell f(x)g(y)h(z)\,dz\,dy\,dx$$

$$= \int_a^b \int_c^d f(x)g(y)\left[\int_k^\ell h(z)\,dz\right] dy\,dx$$

$$= \int_k^\ell h(z)\,dz \cdot \int_a^b \int_c^d f(x)g(y)\,dy\,dx$$

$$= \int_k^\ell h(z)\,dz \cdot \int_a^b f(x)\left[\int_c^d g(y)\,dy\right] dx$$

$$= \int_k^\ell h(z)\,dz \cdot \int_c^d g(y)\,dy \cdot \int_a^b f(x)\,dx$$

$$= \left[\int_a^b f(x)\,dx\right] \cdot \left[\int_c^d g(y)\,dy\right] \cdot \left[\int_k^\ell h(z)\,dz\right]$$

as needed.

# Section 15.8

[LHS]

(40) (a) $$\begin{vmatrix} a_1 & b_1 \\ c_1 & d_1 \end{vmatrix} \begin{vmatrix} a_2 & b_2 \\ c_2 & d_2 \end{vmatrix} = (a_1d_1 - b_1c_1)(a_2d_2 - b_2c_2)$$

$$= a_1a_2d_1d_2 - a_1b_2c_2d_1 - a_2b_1c_1d_2 + b_1b_2c_1c_2$$

[RHS]

$$\begin{vmatrix} a_1a_2 + b_1c_2 & a_1b_2 + b_1d_2 \\ c_1a_2 + d_1c_2 & c_1b_2 + d_1d_2 \end{vmatrix} = (a_1a_2 + b_1c_2)(c_1b_2 + d_1d_2) - (c_1a_2 + d_1c_2)(a_1b_2 + b_1d_2)$$

40 (a) cont'd

$$= \cancel{a_1a_2c_1b_2} + a_1a_2d_1d_2 + b_1b_2c_1c_2 + \cancel{b_1c_2d_1d_2}$$
$$- \cancel{a_1a_2c_1b_2} - a_1b_2c_2d_1 - a_2b_1c_1d_2 - \cancel{b_1c_2d_1d_2}$$

$$= a_1a_2d_1d_2 - a_1b_2c_2d_1 - a_2b_1c_1d_2 + b_1b_2c_1c_2$$

as needed.

(b)
$$\frac{\partial(x,y)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$$

from (a) →
$$= \begin{vmatrix} x_u u_x + x_v v_x & x_u u_y + x_v v_y \\ y_u u_x + y_v v_x & y_u u_y + y_v v_y \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial x}{\partial x} & \frac{\partial x}{\partial y} \\ \frac{\partial y}{\partial x} & \frac{\partial y}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 1$$ as needed.