MATH 3C

CH. 16 HW

Section 16.3

(30) $\vec{F}(x,y) = \frac{c}{(x^2+y^2)^{3/2}}\langle x,y\rangle$

(a) Show $\vec{F}$ conservative.

$$f_y = -3cxy(x^2+y^2)^{-5/2} \quad \& \quad g_x = -3cxy(x^2+y^2)^{-5/2}$$

Since $f_y = g_x$, we have $\vec{F}$ conservative everywhere except at $(0,0)$.

Find $\phi(x,y)$.

$$\phi(x,y) = \int x(x^2+y^2)^{-3/2}\,dx = -c(x^2+y^2)^{-1/2} + c_1(y)$$

&

$$\phi(x,y) = \int y(x^2+y^2)^{-3/2}\,dy = -c(x^2+y^2)^{-1/2} + C_2(x)$$

$$\Rightarrow \boxed{\phi(x,y) = -\frac{c}{\sqrt{x^2+y^2}}}$$

(b) Note: $\vec{F}$ conservative since curl $\vec{F} = \vec{0}$

Find $\phi(x,y,z)$.

$$\phi(x,y,z) = \int cx(x^2+y^2+z^2)^{-3/2}\,dx = -c(x^2+y^2+z^2)^{-1/2} + c_1(y,z)$$

&

$$\phi(x,y,z) = \int cy(x^2+y^2+z^2)^{-3/2}\,dy = -c(x^2+y^2+z^2)^{-1/2} + C_2(x,z)$$

&

$$\phi(x,y,z) = \int cz(x^2+y^2+z^2)^{-3/2}\,dz = -c(x^2+y^2+z^2)^{-1/2} + C_3(x,y)$$

$$\Rightarrow \boxed{\phi(x,y,z) = -\frac{c}{\sqrt{x^2+y^2+z^2}}} \quad (\text{for } \mathbb{R}^3 \setminus \{(0,0,0)\})$$

16.3 cont'd

(34) For Theorem 16.3.2, prove (b) $\Rightarrow$ (c).

Proof: Let C_1, C_2 be 2 piecewise smooth oriented curves in D from $P \to Q$.

Consider the closed curve consisting of C_1 & $-C_2$.

From (b): $$\int_{C_1} \vec{F} \cdot d\vec{r} + \int_{-C_2} \vec{F} \cdot d\vec{r} = 0$$

$$\Rightarrow \int_{C_1} \vec{F} \cdot d\vec{r} = - \int_{-C_2} \vec{F} \cdot d\vec{r}$$

$$\Rightarrow \int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$$

$\Rightarrow \int_C \vec{F} \cdot d\vec{r}$ is independent of path from P to Q. (as needed)

Section 16.7

(18) $\vec{F}(x,y,z) = \langle a, b, c \rangle$ (constant vector field)
& σ: surface of solid G

Using Divergence Theorem: $\operatorname{div} \vec{F} = 0$

$$\Rightarrow \Phi = \iiint_G 0 \, dV = 0$$ (the flux of $\vec{F}$ across σ is zero)

physical explanation: the vector field is constant, so there's no change between what enters & leaves G, thus the flux is zero