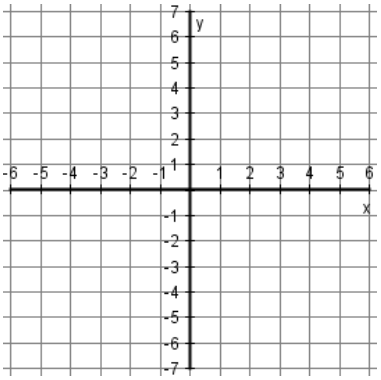
MATH 3C EXAM 1, SPRING 2009 **(J. COCHRANE)**

1. (1.7) Sketch the curve by eliminating the parameter. Include the orientation of the curve.

$x = 2\sin t, \quad y = 3\cos t, \quad 0 \le t \le \dfrac{\pi}{2}$

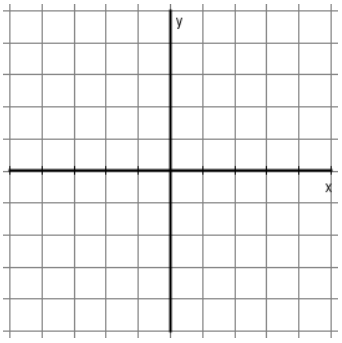


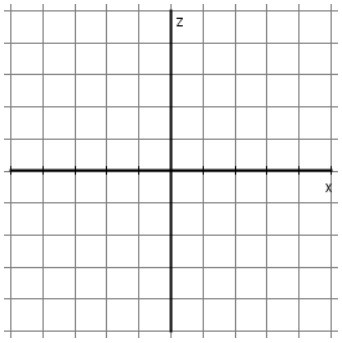
2. (12.6) Find the equation for the plane passing through the point (2, 1, 5) and containing the line $L: \begin{cases} x = -1 + 3t \\ y = -2t \\ z = 2 + 4t \end{cases}$. Write the equation in *point-normal form*.

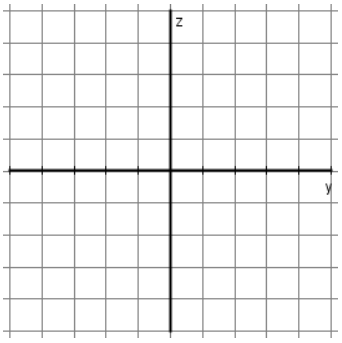
3. (12.1) Write the equation of the sphere with center $(2, -3, 5)$ that is tangent to the *xy*-plane.

4. (12.7) Given the equation $y - x^2 - \dfrac{z^2}{4} = 0$:

(a) Sketch the trace of the surface in each of the 3 coordinate planes. Identify the equation and the location of vertices and asymptotes on each graph.







(b) Sketch the surface in $\mathbb{R}^3$. Circle the type of surface represented by the equation.

ELLIPSOID HYPERBOLOID OF 1 SHEET ELLIPTIC CONE

HYPERBOLIC PARABOLOID HYPERBOLOID OF 2 SHEETS ELLIPTIC PARABOLOID

5. (12.3) Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors in $\mathbb{R}^2$. Let k be a scalar in $\mathbb{R}$.
Prove: $k(\mathbf{u} \bullet \mathbf{v}) = \mathbf{u} \bullet (k\mathbf{v})$

6. (12.5) Determine whether the lines $L_1 : \langle x, y, z \rangle = \langle 3, 5, -1 \rangle + t\langle -1, 3, -4 \rangle$ and $L_2 : \begin{cases} x = 8 + 2t \\ y = -6 - 4t \\ z = 5 + t \end{cases}$ are parallel, intersecting or skew. If the lines intersect, find the point of intersection.

7. (12.3, 12.4) State the geometric significance of each of the following statements for nonzero vectors **u** and **v** in $\mathbb{R}^3$.

(a) $\mathbf{u} \bullet \mathbf{v} = 0$

(b) $\mathbf{u} \times \mathbf{v} = \mathbf{0}$

(c) $\|\mathbf{u} \times \mathbf{v}\| = \mathbf{u} \bullet \mathbf{v}$

8. (12.3) A box is dragged along the floor by a rope that applies a force of 50 lb at an angle of 60° with the floor. How much work is done in moving the box 15 feet? <u>*Label*</u> the answer with appropriate units.

9. (12.8) The equation of a surface is given in spherical coordinates.

$\rho = 2\csc\phi$

(a) Convert the equation to cylindrical coordinates.

(b) Convert the equation to rectangular coordinates.

(c) Sketch the surface in $\mathbb{R}^3$.

10. (11.2) Consider the parametric curve $x = t - 2\cos t,\ y = 2 - 2\sin t$.

(a) Find a formula for $\dfrac{dy}{dx}$.

(b) Find a formula for $\dfrac{d^2y}{dx^2}$ and simplify.

(c) Determine all values of $t \in [0, 2\pi]$ where the curve has a horizontal tangent.