MATH 3C EXAM 2, SPRING 2009 **(J. COCHRANE)**

1. (14.2) Let $f(x,y) = \dfrac{x^2}{x^2 + y^2}$.

(a) Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2 + y^2}$ along the x-axis.

(b) Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2 + y^2}$ along the line $y = x$.

(c) Does the limit of f exist as $(x,y)\to(0,0)$? <u>*Explain*</u>.

2. (13.2) Let $\mathbf{r}(t) = \langle x(t), y(t)\rangle$ be a differentiable vector-valued function. Let $f(t)$ be a differentiable real-valued function.

Prove: $\dfrac{d}{dt}\left[f(t)\mathbf{r}(t)\right] = f(t)\mathbf{r}'(t) + f'(t)\mathbf{r}(t)$

3. (13.1, 13.2, 13.4, 13.5) Let $\mathbf{r}(t) = (1 + 2\sin t)\mathbf{i} + (3 - 2\cos t)\mathbf{j}$; $0 \le t \le 2\pi$.

(a) Sketch $\mathbf{r}(t)$. Include the orientation of the curve.

(b) Calculate $\mathbf{r}\left(\dfrac{\pi}{3}\right)$.

(c) Sketch the vector $\mathbf{r}\left(\dfrac{\pi}{3}\right)$ in standard position on the graph above. Label it with ***c***.

(d) Calculate $\mathbf{r}'\left(\dfrac{\pi}{3}\right)$.

(e) Sketch the vector $\mathbf{r}'\left(\dfrac{\pi}{3}\right)$ in translated position on the graph above, with its tail at the tip of $\mathbf{r}\left(\dfrac{\pi}{3}\right)$. Label it with ***e***.

(f) Does $\mathbf{r}'\left(\dfrac{\pi}{3}\right) = \mathbf{T}\left(\dfrac{\pi}{3}\right)$? <u>*Explain*</u>.

(g) Find an equation for the tangent line to the curve at $t = \dfrac{\pi}{3}$. Write the equation in <u>*vector form*</u>.

(h) Without calculating the actual values, compare the curvatures $\kappa(0)$ and $\kappa\left(\dfrac{\pi}{2}\right)$.

4. (14.3) Let $f(x,y) = y\ln\left(x+y^2\right) + y^2\sqrt{x}$.

(a) Find the slope of the surface in the *x*-direction at the point $\left(1,1,1+\ln 2\right)$.

(b) Find $\dfrac{\partial^2 f}{\partial y \partial x}$.

5. (13.5) Find the **maximum** value of the radius of curvature for the curve
$\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \sin t\mathbf{k};\quad 0 \le t < 2\pi$.

6. (13.3) Find parametric equations for $\mathbf{r}(t) = \langle 6\sin 2t,\ 6\cos 2t \rangle$ using arc length, *s*, as a parameter. Use the point on the curve where $t = 0$ as the reference point.

7. (13.6) Suppose that $\mathbf{r}(t)$ is the *position function* of a particle moving in 2-space or 3-space. For each part, explain, in words, what the given quantity represents physically.

(a) $\left\|\mathbf{r}(t)\right\|$ (b) $\left\|\mathbf{r}'(t)\right\|$ (c) $\int_a^b \left\|\mathbf{r}'(t)\right\| dt$

8. (14.1) Let $f(x,y) = \sqrt{16 - x^2 - y^2}$.

(a) Sketch the domain of *f*. Remember to ***label*** the axes.

(b) Sketch the graph of *f*. Remember to ***label*** the axes. You may supplement your picture with words to clarify what you are trying to draw.

9. (13.6) An object's acceleration is given as a function of time by
$\mathbf{a}(t) = \dfrac{1}{(t+1)^2}\mathbf{i} - e^{-2t}\mathbf{k}$. If the object's initial velocity (when $t = 0$) is $\langle 3,-1,0 \rangle$, find a formula for the object's velocity as a function of time.

10. (13.4) Let $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \mathbf{k}$. At $t = \dfrac{\pi}{4}$, we have $\mathbf{T}\left(\dfrac{\pi}{4}\right) = \left\langle -\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}, 0 \right\rangle$ and
$\mathbf{N}\left(\dfrac{\pi}{4}\right) = \left\langle -\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}, 0 \right\rangle$.

(a) Find $\mathbf{B}\left(\dfrac{\pi}{4}\right)$, the binormal vector at $t = \dfrac{\pi}{4}$.

(b) Find an equation for the *normal* plane (also called the **NB**-plane) at the point corresponding to $t = \dfrac{\pi}{4}$.