MATH 3C EXAM 3, SPRING 2009 **(J. COCHRANE)**

1. (15.4) Find the surface area of the portion of the paraboloid $\mathbf{r}(u,v) = \langle u\cos v, u\sin v, u^2 \rangle$ for which $0 \le u \le 1,\ 0 \le v \le \pi$.

2. (15.2) Consider $\int_0^2 \int_1^{e^y} f(x,y)\,dx\,dy$.

(a) Sketch the region R of integration.

(b) Express the integral as an equivalent integral with the order of integration reversed.

(c) If $f(x,y) = 1$, this integral represents ______________________.

(d) If $f(x,y) \neq k$, (k constant) this integral represents ______________________.

3. (14.6) *Explain*, in words.

(a) How are the directional derivative and the gradient of a function related?

(b) In what direction does the directional derivative of a differentiable function have its maximum value?

4. (14.5) Given $f(x,y) = x^2y^3 + x\sin y$ and $\mathbf{r}(t) = \langle t^2, e^{3t} \rangle$, use the chain rule to calculate $\frac{d}{dt}\left[f(\mathbf{r}(t)) \right]$. You must use the chain rule to receive credit for the answer.

5. (15.8) Use the transformation $u = xy,\ v = xy^4$ to find $\iint_R \sin(xy)\,dA$ where R is the region enclosed by the curves $xy = \pi,\ xy = 2\pi,\ xy^4 = 1,\ xy^4 = 2$.

6. (15.6, 15.3) Find the *mass* of the lamina in the first quadrant that is inside $x^2 + y^2 = 8x$ and outside $x^2 + y^2 = 16$ if the density of the region is $\delta(x,y) = \dfrac{y}{\sqrt{x^2+y^2}}$.

7. (14.7) (a) Find an equation of the tangent plane to the surface $xz - yz^3 + yz^2 = 2$ at the point $(2, -1, 1)$.

(b) Find a point on the surface $z = 3x^2 - y^2$ at which the tangent plane is parallel to the plane $6x + 4y - z = 5$.

8. (15.5, 15.7) Let *G* be the region bounded above by the sphere $\rho = 2$ and below by the cone $\phi = \frac{\pi}{3}$. Set up, but DO NOT EVALUATE, the volume of *G* as an iterated integral in:

(a) spherical coordinates

(b) cylindrical coordinates

(c) rectangular coordinates

9. (15.1) Give a geometric argument to show that $0 < \int_0^{\pi}\int_0^{\pi} \sin\sqrt{xy}\,dy\,dx \le \pi^2$. DO NOT EVALUATE THE INTEGRAL.

10. (14.8, 14.9) Let $f(x, y) = x^2 y^2$ and let $D = \{(x, y) | 4x^2 + y^2 \le 8\}$.

(a) The ______________________ theorem ensures that *f* achieves an absolute max and absolute min on *D*, since *f* is ___________________ and *D* is __________________ and ____________________.

(b) Find the <u>*location*</u> and <u>*value*</u> of the absolute max and min of *f* on *D*. Use the method of Lagrange multipliers on the boundary.