Ratios and Proportions

The ratio of $a$ to $b$ is $\frac{a}{b}$ ($b \neq 0$).

A proportion is an equation of two ratios.

Example 1:

\[
\frac{x}{4} = \frac{7}{6}
\]

Method 1: (The “Heart” Method)

\[
\frac{x}{4} = \frac{7}{6}
\]

\[
6 \cdot x = 4 \cdot 7
\]

\[
\frac{6x}{6} = \frac{28}{6}
\]

\[
x = \frac{28}{6}
\]

We must reduce the fraction.

\[
x = \frac{14}{3}
\]

**NOTE:** The Heart Method (sometimes referred to as cross-multiplication) **ONLY** works for PROPORTIONS!
Method 2: (Clearing the Fractions)
Find the LCD.

\[ \text{LCD} = \ldots \]

\[ 12 \left( \frac{x}{4} \right) = 12 \left( \frac{7}{6} \right) \]

Multiply both sides by the LCD

\[ \frac{3x}{3} = \frac{14}{3} \]
Reduce.

\[ x = \frac{14}{3} \]

**NOTE:** Clearing the fractions ALWAYS WORKS!!!

Example 2:

\[ \frac{x + 1}{3} = \frac{x}{2} \]

The Heart Method:

\[ 2(x + 1) = 3 \cdot x \]

\[ 2x + 2 = 3x \]

\[ -2x - 2x \]

\[ 2 = x \]

So \( x = 2 \) is the solution.
Clearing the Fractions:
Find the LCD.

\[
\text{LCD} = \underline{\quad}.\]

\[
6 \left( \frac{x+1}{3} \right) = 6 \left( \frac{x}{2} \right)
\]

\[
2(x + 1) = 3x
\]

\[
2x + 2 = 3x
\]

\[
-2x - 2x
\]

\[
2 = x
\]

OR \( x = 2 \)

Example 3:

\[
\frac{4-z}{3} = \frac{3z-2}{5}
\]

Using the Heart Method, we get

\[
5(4 - z) = 3(3z - 2)
\]

\[
20 - 5z = 9z - 6
\]

\[
+5z + 5z
\]

\[
20 = 14z - 6
\]

\[
+6 + 6
\]

\[
\frac{26}{14} = \frac{14z}{14}
\]

\[
z = \frac{26}{14}
\]

\[
z = \frac{13}{7}
\]

Reduce.
Similar Triangles

Two triangles are said to be similar if the corresponding angles are the same measurement.

For similar triangles, the ratios of corresponding sides are equal.

\[
\frac{a}{c} = \frac{b}{e} = \frac{c}{f}
\]

Example 4:

Solve for \(x\) given that the pair of triangles are similar.

We can set up a proportion to solve for \(x\).

\[
\frac{x}{5} = \frac{1}{3}
\]

Using the Heart Method, we get

\[
3x = 5
\]

Solve for \(x\), by dividing by 3.

\[
x = \frac{5}{3}
\]
Ratios and Proportions

Practice Problems

Solve each equation.

1. \[ \frac{x}{4} = \frac{2}{3} \]

2. \[ \frac{x+2}{6} = \frac{x+4}{4} \]

3. Solve for \( x \) given that the pair of triangles are similar.