Slope of a Line

Consider the equation

\[ y = 2x - 4 \]

We can make an \( x, y \) chart to help graph the line represented by this equation.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Do you notice a pattern between the change of \( y \) values and the change of \( x \) values?

This pattern is referred to as the slope of the line.

The slope of a line is the ratio of the change in \( y \) (vertical change) to the change in \( x \) (horizontal change). We use the letter \( m \) to represent slope.

\[
m = \frac{\text{change of } y}{\text{change of } x}
\]
Let’s look at:

\[ y = 2x - 4 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1</td>
<td>-2</td>
</tr>
<tr>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>+1</td>
<td>2</td>
</tr>
<tr>
<td>+1</td>
<td>4</td>
</tr>
</tbody>
</table>

The \( x \) values increase by 1

The \( y \) values increase by 2

\[
m = \frac{\text{change of } y}{\text{change of } x} = \frac{2}{1} = 2
\]

dependently, \( m = 2 \) for the line \( y = 2x - 4 \).

We can use the slope to help graph the line represented by the equation \( y = 2x - 4 \).
Looking left to right:

- **Positive**
  - If the line is going up, the line has a **negative** slope.

- **Negative**
  - If the line is going down, the line has a **positive** slope.

- **Slope is zero**
  - All horizontal lines have a slope equal to **zero**.

- **Undefined slope**
  - All vertical lines have an **undefined** slope.
Finding the slope of a line given two points on the line:

Let's call \( P_1 \) (point 1) \((x_1, y_1)\) and \( P_2 \) (point 2) \((x_2, y_2)\)

\[
m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

**Example 1**

Find the slope of the line that passes through the points \((6, 2)\) and \((18, 8)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 2}{18 - 6} = \frac{6}{12} = \frac{1}{2}
\]

*reduce*
Parallel Lines:
Two lines are parallel if they never intersect. Parallel lines have the same slope.

Perpendicular Lines: Two lines are perpendicular if and only if they intersect at a 90° angle. The slopes of two perpendicular lines are negative reciprocals of each other.

Finding the slope using the equation of a line:

Recall: Equation of a line
\[ y = mx + b \]

Step 1: Solve the equation for \( y \)
Step 2: Identify the coefficient of \( x \) — this is the slope of the line.

<table>
<thead>
<tr>
<th>( m_1 )</th>
<th>( m_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-\frac{1}{3}</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>-\frac{2}{3}</td>
<td>\frac{2}{3}</td>
</tr>
<tr>
<td>0</td>
<td>Undefined</td>
</tr>
</tbody>
</table>
Example 2

Find the slope of the line $2x + y = 8$

**Step 1:**

\[
\begin{align*}
2x + y &= 8 \\
-2x &- 2x \\
y &= -2x + 8
\end{align*}
\]

**NOTE:** we prefer to see the $x$ before the constant 

**Step 2:** The coefficient of $x$ is $-2$

Therefore, the slope of the line $2x + y = 8$ is $-2$.

**NOTE:** Any line parallel to the line $2x + y = 8$ has a slope of $-2$ and any line perpendicular to the line has a slope of $\frac{1}{2}$.
Example 3

Identify the lines as being parallel, perpendicular or neither:

\[ x + y = 4 \]
\[ 3x + 3y = 6 \]

First we find the slope of each line.

**Step 1: Solve for \( y \) (for both Line 1 and Line 2)**

**Line 1:**
\[
\begin{align*}
\quad x + y &= 4 \\
-x & \quad -x \\
\hline
y &= -x + 4
\end{align*}
\]

Slope of line 1: \( m_1 = -1 \)

**Line 2:**
\[
\begin{align*}
\quad 3x + 3y &= 6 \\
-3x & \quad -3x \\
\hline
3y &= -3x + 6 \\
\frac{3}{3} & \quad \frac{-3x}{3} \quad \frac{6}{3} \\
\hline
y &= -x + 2
\end{align*}
\]

Slope of line 2: \( m_2 = -1 \)

Therefore, these two lines are parallel.
1. Find the slope of the line that passes through the points \((5, 7)\) and \((-1, 3)\).

2. Match each equation with its corresponding graph:
   - a) \(x + y = 1\)
   - b) \(-x + y = 1\)
   - c) \(y = 1\)
   - d) \(x = 1\)