APPLICATIONS OF QUADRATIC EQUATIONS

= Integer Problems

Recall: 
- Consecutive integers are of the form
  \[ x, x+1, x+2, \ldots \]
- Consecutive odd integers AND
- Consecutive even integers are of the form
  \[ x, x+2, x+4, \ldots \]

Example 1: Find two consecutive integers whose product is 11 more than their sum.

We must translate into math terms:

"two consecutive integers" \( \rightarrow \) \( x \) & \( x+1 \)

"product" \( \rightarrow \) multiplication

"is" \( \rightarrow \) =

"11 more than" \( \rightarrow \) + 11

"sum" \( \rightarrow \) addition

So our equation is

\[ x(x+1) = x + (x+1) + 11 \]

Now we solve for \( x \):

- Simplify each side first:
  \[ x^2 + x = 2x + 12 \]
- Set equal to zero:
\[
x^2 + x = 2x + 12
\]
\[
-2x - 12 = -2x - 12
\]
\[
x^2 + x - 2x - 12 = 0
\]
\[
x^2 - x - 12 = 0
\]

- **Factor:**

  \[
a = 1
\]

  \[
b = -1
\]

  \[
c = -12
\]

  \[
a \cdot c = -12
\]

  **sum** (we want -1)

  \[
  \begin{array}{ccc}
  1 & -12 & -11 \\
  2 & -6 & -4 \\
  4 & -3 & 1 \\
  -4 & 3 & -1 \\
  \end{array}
\]

  since \(a = 1\), we can use the shortcut

  \[
  (x - 4)(x + 3) = 0
\]

  - set each factor equal to zero

  \[
  x - 4 = 0 \quad x + 3 = 0
\]

  - solve for \(x\):

  \[
  \begin{align*}
  x - 4 &= 0 \\
  +4 \quad +4 & \quad -3 \quad -3 \\
  x &= 4 \\
  x &= -3
  \end{align*}
\]
We can check to see if both values we got solve the word problem:

- $x = 4$
  - If $x = 4$, the two consecutive integers are 4 and 5
  - Product: $4 \cdot 5 = 20$
  - Sum: $4 + 5 = 9$
  - Is the product 11 more than the sum?
    - $20 = 9 + 11$
    - Yes! so $x = 4$ is an answer.

- $x = -3$
  - If $x = -3$, the two consecutive integers are -3 and -2
  - Product: $(-3)(-2) = 6$
  - Sum: $(-3) + (-2) = -5$
  - Is the product 11 more than the sum?
    - $6 = -5 + 11$
    - Yes! so $x = -3$ is also an answer.
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Find two consecutive integers whose product is 1 more than their sum.