Simplifying Rational Expressions

Consider the expression:

\[
\frac{2x+4}{2}
\]

We CANNOT cancel the 2's, but we can factor the numerator

\[
2x + 4 = 2(x + 2)
\]

so our expression can be written

\[
\frac{2(x+2)}{2}
\]

Now the 2 in the numerator can be cancelled with the 2 in the denominator.

\[
\frac{2(x+2)}{2} = x + 2
\]

Example 1:

Simplify each expression:

a.) \[
\frac{3x+12}{2x+8}
\]

We must factor the top and bottom first to see if any factors will cancel:

\[
\frac{3x+12}{2x+8} = \frac{3(x+4)}{2(x+4)} = \frac{3}{2}
\]

Notice that the binomials cancel!
b.) \( \frac{(x+3)(x+5)}{(x-3)(x+5)} \)

Here the numerator and denominator are already factored, since there is an \((x + 5)\) on the TOP AND in the BOTTOM, they will cancel.

\[
\frac{(x+3)(x+5)}{(x-3)(x+5)} = \frac{(x+3)(x+5)}{(x-3)(x+5)} = \frac{x+3}{x-3}
\]

c.) \( \frac{4x^2-4x}{5x-5} \)

We need to factor the top and the bottom first, to see if any factors will cancel:

\[
\frac{4x^2-4x}{5x-5} = \frac{4x(x-1)}{5(x-1)} = \frac{4x}{5}
\]

d.) \( \frac{x^2+2x-15}{x^2+6x+5} \)

We need to factor the top and bottom first to see if any factors will cancel.

Let’s start with the top:

\[
x^2 + 2x - 15
\]

We need to use the \textit{abc} method.
\[ x^2 + 2x - 15 \]
\[ a = 1 \]
\[ b = 2 \]
\[ c = -15 \]
\[ a \cdot c = -15 \] we need factors of \(-15\) that add to 2 (the \(b\)-term)

<table>
<thead>
<tr>
<th>(a)</th>
<th>(c)</th>
<th>(bc)</th>
<th>Sum (we want 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-15</td>
<td>-14</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-5</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td>5</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

So we get:
\[ x^2 + 2x - 15 = (x - 3)(x + 5) \]

Now the denominator:
\[ x^2 + 6x + 5 \]
\[ a = 1 \]
\[ b = 6 \]
\[ c = 5 \]
\[ a \cdot c = 5 \]
We need factors of 5 that add to 6 (the \(b\)-term).
The factors are 5 and 1.
So we get:
\[ x^2 + 6x + 5 = (x + 5)(x + 1) \]

Therefore our expression can be written:
\[ \frac{x^2 + 2x - 15}{x^2 + 6x + 5} = \frac{(x - 3)(x + 5)}{(x + 5)(x + 1)} = \]

Will anything cancel?
What is the final solution?

Answer:
e.) 
\[
\frac{6 - x}{x - 6}
\]

We are hoping to factor the top or bottom so that something will cancel.

If we factor out an \(-1\) from the top, we get:

\[
\frac{6 - x}{x - 6} = \frac{-1(-6 + x)}{x - 6}
\]

Using the commutative property of addition, we can write the top binomial as

\[
x + (-6) = x - 6
\]

So our expression becomes:

\[
\frac{-1(x - 6)}{x - 6}
\]

The binomials cancel and our final answer is \(-1\).
Simplifying Rational Expressions

Practice Problems

Simplify each expression:

1. \( \frac{5x - 15}{x - 3} \)

2. \( \frac{7x^2 + 7x}{2x + 2} \)

3. \( \frac{x^2 - 9}{x^2 + x - 6} \)

4. \( \frac{x^2 - 1}{1 - x} \)