Multiplying and Dividing Rational Expressions

Recall: \( \frac{3}{4} \div \frac{7}{8} = \frac{3 \cdot 8}{4 \cdot 7} = \frac{3 \cdot 8^2}{4 \cdot 7^2} = \frac{3 \cdot 8}{4 \cdot 7} = \frac{6}{7} \)

Example 1:

A) \( \frac{x}{4} \div \frac{7}{6} \)
\[ \frac{x}{4} \div \frac{7}{6} = \frac{x \cdot 6}{4 \cdot 7} = \frac{2x}{4 \cdot 7} = \frac{2x}{28} = \frac{x}{7} \]

B) \( \frac{x+3}{x-2} \div \frac{x-1}{x-2} \)
\[ \frac{x+3}{x-2} \div \frac{x-1}{x-2} = \frac{x+3 \cdot (x-2)}{x-1} \cdot \frac{1}{(x-2)} = \frac{x+3}{x-1} \]

C) \( \frac{(x+2)^2}{x+1} \div \frac{x-1}{x+2} \)
\[ \frac{(x+2)^2}{x+1} \div \frac{x-1}{x+2} = \frac{(x+2)^2 \cdot (x+2)}{x+1 \cdot (x-1)} = \frac{(x+2)^2}{x+1 \cdot (x-1)} \]
Here the binomial $x+2$ in the 
bottom will cancel with one of 
the $x+2$'s in the top.

If we write it out we get 

$$\frac{(x+2)^2 \cdot x-1}{x+1} = \frac{(x+2)(x+2)}{x+1} \cdot \frac{x-1}{x+2} = \frac{(x+2)(x-1)}{x+1}$$

Steps for multiplying and dividing rational 
expressions:

**Step 1:** Make all division problems 
into multiplication problems 
(see example 1-A)

**Step 2:** Factor the numerator and 
denominator completely.

**Step 3:** Cancel out all factors that 
are in the numerator AND 
the denominator (reduce)

Note: Keep your final answer in factored 
form – you do not need to multiply 
or distribute terms.
Example 2:
\[ \frac{x^2 + 6x + 9}{x^2 - 4} \cdot \frac{x - 2}{x + 3} \]

Step 1: This step is not needed since it is already a multiplication problem.

Step 2: Factor.
\[ \frac{x^2 + 6x + 9}{x^2 - 4} \cdot \frac{x - 2}{x + 3} = \frac{(x+3)(x+3)}{(x+2)(x-2)} \cdot \frac{(x-2)}{(x+3)} \]

Step 3: Cancel factors that appear on top and bottom.
\[ \frac{(x+3)(x+3)}{(x+2)(x-2)} \cdot \frac{(x-2)}{(x+3)} \]

So our final answer is \( \frac{x+3}{x+2} \)

Example 3:
\[ \frac{x^2 + 6x + 8}{x^2 - x - 6} \div \frac{x^2 + 3x - 4}{x^2 + 2x - 3} \]

Step 1: \[ \frac{x^2 + 6x + 8}{x^2 - x - 6} \div \frac{x^2 + 3x - 4}{x^2 + 2x - 3} \]
\[ \frac{x^2 + 6x + 8}{x^2 - x - 6} \cdot \frac{x^2 + 2x - 3}{x^2 + 3x - 4} \]
Step 2: Factor
\[ \frac{x^2 - 6x + 8}{x^2 + 2x - 3} = \frac{(x+2)(x+4)}{(x+3)(x-1)} \]
\[ \frac{x^2 - x - 6}{x^2 + 3x - 4} = \frac{(x-3)(x+2)}{(x+4)(x-1)} \]

Step 3: Cancel factors that appear on top and bottom.

\[ \frac{(x+2)(x+4)}{(x-3)(x+2)} \cdot \frac{(x+3)(x-1)}{(x+4)(x-1)} \]

So our final answer is

\[ \frac{x+3}{x-3} \]
Finding the LCD of Rational Expressions Handout

Steps to finding the LCD of Rational Expressions:

Step 1: Factor each denominator completely.

Step 2: List each different factor from step 1, the greatest number of times it appears in a denominator.

Step 3: Multiply the factors from step 2 — this is the LCD.

Example: Find the LCD of the terms

\[ \frac{4}{x}, \frac{3}{x+1}, \frac{2}{x^2+2x+1}, \frac{1}{x^2+3x+2} \]

There are four denominators:

\[ x, x+1, x^2+2x+1, x^2+3x+2 \]

Step 1: Factoring each denominator we get

\[ x, x+1, (x+1)^2, (x+1)(x+2) \]

Step 2: The different factors are \( x, x+1, \) and \( x+2 \)

the highest power of \( x \) is 1,

the highest power of \( (x+1) \) is 2,

and the highest power of \( (x+2) \) is 1

Step 3: \( \text{LCD} = x(x+1)^2(x+2) \)