SOLVING EQUATIONS WITH RATIONAL EXPRESSIONS

TO SOLVE EQUATIONS WITH RATIONAL EXPRESSIONS, IDENTIFY THE LCD, MULTIPLY BOTH SIDES BY THE LCD, THEN SOLVE.

EXAMPLE 1: SOLVE FOR X:

\[ \frac{1}{x-1} + \frac{3}{x-1} = \frac{7}{4} \]

LCM = \(4(x-1)\)

\[ \frac{4(x-1)}{1} \left( \frac{1}{x-1} \right) + \frac{4(x-1)}{1} \left( \frac{3}{x-1} \right) = \frac{4(x-1)(7)}{1} \]

\[ 4 + 4(3) = 7(x-1) \]

\[ 4 + 12 = 7x - 7 \]

\[ 16 = 7x - 7 \]

\[ 16 + 7 = 7x \]

\[ 23 = 7x \]

\[ x = \frac{23}{7} \]

*Note: You could add these two fractions since they have the same denominators.*
Example 2:
\[
\frac{3}{x-1} + \frac{1}{x-4} = \frac{x-2}{x-1}
\]

\[\text{LCD} = (x-1)(x-4)\]

\[
\frac{(x-1)(x-4)}{x-1} \cdot \frac{3}{1} + \frac{(x-1)(x-4)}{x-4} \cdot \frac{1}{1} = \frac{(x-1)(x-4)(x-2)}{(x-1)(x-4)}
\]

\[
3(x-4) + (x-1) = (x-4)(x-2)
\]

\[
3x - 12 + x - 1 = x^2 - 6x + 8
\]

\[
4x - 13 = x^2 - 6x + 8
\]

\[
-4x + 13 = -4x + 13
\]

\[
0 = x^2 - 10x + 21
\]

\[
a = 1
\]

\[
b = -10
\]

\[
c = 21
\]

\[
a \cdot c = 21
\]

\[
\text{Sum}
\]

\[
\begin{array}{ccc}
1 & 21 & 1 \\
3 & 7 & 10 \\
-3 & -7 & -10 \\
\end{array}
\]

\[
(x-3)(x-7) = 0
\]

\[
x - 3 = 0 \quad \text{or} \quad x - 7 = 0
\]

\[
\boxed{x = 3 \quad x = 7}
\]
Solving equations with rational expressions Practice Problems

Solve each equation

1. \( \frac{1}{x-1} + \frac{1}{x-3} = \frac{3}{x-1} \)

2. \( \frac{x+2}{x-1} + \frac{1}{x-5} = \frac{12}{x-1} \)