Extraneous Solutions Handout I

Equations with rational expressions

An extraneous solution to an equation with rational expressions is a solution that causes a denominator to equal zero.

Example: \( \frac{1}{x+4} + \frac{x}{x-4} = \frac{-8}{x^2-16} \)
\[
\frac{1}{x+4} + \frac{x}{x-4} = \frac{-8}{(x+4)(x-4)}
\]
\[
\text{LCD} = (x+4)(x-4)
\]
\[
\frac{(x+4)(x-4)}{(x+4)} \left( \frac{1}{x+4} \right) + \frac{(x+4)(x-4)}{(x-4)} \left( \frac{x}{x-4} \right) = \frac{(x+4)(x-4)}{(x+4)(x-4)} \left( \frac{-8}{(x+4)(x-4)} \right)
\]
\[
(x-4) + x(x+4) = -8
\]
\[
X-4 + x^2 + 4x = -8 \quad \text{← Distribute}
\]
\[
x^2 + 5x - 4 = -8 \quad \text{← Collect like terms}
\]
\[
x^2 + 5x + 4 = 0 \quad \text{← Set equal to zero}
\]
\[
(x+4)(x+1) = 0 \quad \text{← Factor}
\]
\[
x+4=0 \quad x+1=0 \quad \text{← Set each factor equal to zero}
\]
\[
x=-4 \quad x=-1 \quad \text{← Solve for x}
\]

Since the solution \( x=-4 \) causes a denominator to equal zero, we call this an extraneous solution. Therefore the answer is \( x=-1 \).