\[ V_{\text{shell}} = V_{\text{outer shell}} - V_{\text{inner shell}} \]
\[ = \pi r_1^2 h - \pi r_2^2 h \]
\[ = \pi h (r_1^2 - r_2^2) \]
\[ = \pi h (r_1 + r_2)(r_1 - r_2) \]

Taking the limit as \( \Delta x \to 0 \)

\[ V_{\text{shell}} = \lim_{\Delta x \to 0} \left[ \pi h \left( \frac{r_1 + r_2}{2} \right) \frac{(r_1 - r_2)}{\Delta x} \right] \]
\[ \Rightarrow \pi h \cdot 2\pi \cdot \Delta x \]
\[ \therefore V_{\text{shell}} = 2\pi rh \Delta x \]
Example 1: Find the volume of the shape by rotating the bounded region about the indicated axis.

a) \( y = \frac{1}{x^3} \), \( x = 1 \), \( x = 2 \) : Rotated about the line \( x = -1 \)

\[ h = y \]
\[ r = x - (-1) \]
\[ r = x + 1 \]

\[ V_{\text{shell}} = 2\pi \int \Delta x \]
\[ = 2\pi (x+1) \cdot y \cdot \Delta x \]
\[ = 2\pi (x+1) \cdot \frac{1}{x^3} \cdot \Delta x \]

\[ V = 2\pi \int_{1}^{2} \left( \frac{x+1}{x^3} \right) dx \]
b) $y^2 = x$, $y = 1$, $x = 0$ : Rotated about the line $y = 2$

$r = 2 - y$
$h = x = y^2$

$V_{\text{shell}} = 2\pi rh \Delta y$

$V = 2\pi \int_{0}^{1} (2-y)(y^2) \, dy$