"L" is approximately equal to the length of the curve on \([x_{k-1}, x_k]\). The smaller our \(\Delta x\), the closer "L" get to the length of the curve.

\[\Delta x = x_k - x_{k-1}\]

\[
\therefore \text{as } \Delta x \text{ converges to } 0, \quad \Delta x \to 0,
\]

"L" converges to the length of the curve.
Using the Mean Value Theorem, there must exist a value "c" in \((x_{k-1}, x_k)\) such that

\[ f'(c) = \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}} \]

\[ f''(c) \cdot \Delta x = \frac{f(x_k) - f(x_{k-1})}{\Delta y} \]

Since \(L = \sqrt{\Delta x^2 + \Delta y^2}\), we have:

\[ L = \sqrt{\Delta x^2 + [f'(c) \cdot \Delta x]^2} \]

\[ L = \sqrt{\Delta x^2 + [f'(c)]^2 \cdot \Delta x^2} \]

\[ L = \sqrt{\Delta x^2 \left(1 + [f'(c)]^2\right)} \]

\[ L = \sqrt{1 + [f'(c)]^2} \cdot \Delta x \]
Letting $\Delta x \to 0$,

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx$$

Example 1: Find the exact "arc length" of the curve $f(x) = x^{2/3}$ on $[1, 8]$.

$$f(x) = x^{2/3}$$
$$f'(x) = \frac{2}{3} x^{-1/3}$$

$$L = \int_1^8 \sqrt{1 + \left(\frac{2}{3} x^{-1/3}\right)^2} \, dx$$

$$= \int_1^8 \sqrt{1 + \frac{4}{9x^{2/3}}} \, dx$$

$$= \int_1^8 \sqrt{\frac{9x^{2/3} + 4}{9x^{2/3}}} \, dx$$

$$= \int_1^8 \frac{1}{3x^{1/3}} \sqrt{9x^{2/3} + 4} \, dx$$

$$= \frac{1}{3} \int_1^8 \frac{1}{x^{1/3}} \sqrt{9x^{2/3} + 4} \, dx$$

Let $u = 9x^{2/3} + 4$,
$$du = 6x^{-1/3} \, dx$$

$$= \frac{1}{3} \int_1^8 \frac{1}{u^{1/2}} \, du$$

$$= \frac{1}{18} \left[ \frac{2}{3} u^{3/2} \right]_1^8$$

$$= \frac{1}{27} \left[ u^{3/2} \right]_1^8$$

$$= \frac{1}{27} \left[ 40 - 13 \right]^{3/2}$$

$$= \frac{1}{27} \left[ (2\sqrt{10})^3 - (\sqrt{13})^3 \right]$$

$$= \frac{80\sqrt{10} - 13\sqrt{13}}{27}$$