Theorem 3.8.1 Rolle's Theorem
Let $f$ be continuous on $[a, b]$ and differentiable on $(a, b)$.
If $f(a) = 0$ and $f(b) = 0$, then there must be one point $c$, such that $f'(c) = 0$

\[ f'(c) = 0 \]

Theorem 3.8.2 The Mean Value Theorem
Let $f$ be continuous on $[a, b]$ and differentiable on $(a, b)$. Then there is at least one point $c$ in $(a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

\[ m_{sec} = \frac{f(b) - f(a)}{b - a} \]
\[ m = f'(c) \]
# 1: Verify Rolle's theorem with \( f(x) = x^2 - 8x + 15 \) on \([3, 5]\).

\[
\begin{align*}
f(3) &= 3^2 - 8(3) + 15 = 0 \\
f(5) &= 5^2 - 8(5) + 15 = 0
\end{align*}
\]

To find \( c \), we find \( f'(x) \) and set it equal to zero.

\[
f'(x) = 2x - 8
\]

\[
\begin{align*}
2x - 8 &= 0 \\
2x &= 8 \\
x &= 4
\end{align*}
\]

\[
\therefore \ c = 4 \quad \text{which is in } [3, 5] \quad \text{and } f'(4) = 0
\]

# 7 Verify the Mean Value theorem with \( f(x) = \sqrt{25-x^2} \) on \([-5, 3]\).

1. \( f(3) = \sqrt{25-9} = 4 \)

\[
f(-5) = \sqrt{25-25} = 0
\]

2. \( \frac{f(b)-f(a)}{b-a} = \frac{4-0}{3-(-5)} = \frac{4}{8} = \frac{1}{2} \)

3. To find \( c \), we find \( f''(x) \) and set it equal to \( \text{m}_{\text{sec}} = \frac{1}{2} \).

\[
f'(x) = \frac{1}{2} \left( 25 - x^2 \right)^{-\frac{1}{2}} \cdot (-2x)
\]

\[
\Rightarrow \quad \frac{1}{2} = - \frac{x}{\sqrt{25-x^2}}
\]

\[
\begin{align*}
\sqrt{25-x^2} &= -2x \\
5 &= x^2 \\
25-x^2 &= 4x^2 \\
\sqrt{5} &= x
\end{align*}
\]

\[
25 = 5x^2
\]
To verify the value of $c$, we evaluate $f''(x)$ at $x = \pm \sqrt{5}$

\[
f''(x) = -\frac{x}{\sqrt{25-x^2}}
\]

\[
f''(-\sqrt{5}) = -\frac{-\sqrt{5}}{\sqrt{25-(-\sqrt{5})^2}} = \frac{\sqrt{5}}{\sqrt{20}} > \frac{\sqrt{5}}{2\sqrt{5}} > \frac{1}{2}
\]

\[\therefore c = -\sqrt{5}\]