SECTION: OVERVIEW OF THE AREA PROBLEM:

- **Example 1**: The Area Problem

  approximate area under the curve \( f(x) = x^2 \) on interval \([0, 1]\)

  \[
  A_{\text{rectangle}} = l \cdot w
  \]

  Here we will use 4 rectangles to create 4 partitions; each having a width of \( \frac{1}{4} \).

  We will use the right-hand endpoints method to define the height of each rectangle.

  \[
  \begin{align*}
  f\left(\frac{1}{4}\right) &= \frac{1}{16} \\
  f\left(\frac{1}{2}\right) &= \frac{1}{4} \\
  f\left(\frac{3}{4}\right) &= \frac{9}{16} \\
  f(1) &= 1
  \end{align*}
  \]

  The height of the rectangles

  approximated area using these 4 rectangles:

  \[
  \text{Area} = \frac{1}{4}\left(\frac{1}{16}\right) + \frac{1}{4}\left(\frac{1}{4}\right) + \frac{1}{4}\left(\frac{9}{16}\right) + \frac{1}{4}(1)
  \]

  \[
  = \frac{1}{64} + \frac{1}{16} + \frac{9}{64} + 1
  \]

  \[
  = \frac{32}{64} = \frac{15}{32} \approx 0.4688
  \]

  \[
  \therefore \text{area under } f(x) = x^2 \text{ is approximately } \frac{15}{32} \approx 0.4688
  \]

  ACTUAL AREA = \( \frac{1}{3} \)

- **Example 2**:

  estimate the area under \( f(x) = \frac{1}{x} \) on \([1, 2]\) with \( n = 5 \)

  **Note:** \( n \) represents the number of partitions (rectangles). Use the right-hand method.
**Example 2: Continues:**

Base of each rectangle is \( \frac{1}{5} \)

\[
\begin{align*}
&f \left( \frac{4}{5} \right) = \frac{5}{6} \\
&f \left( \frac{3}{5} \right) = \frac{5}{4} \\
&f \left( \frac{2}{5} \right) = \frac{5}{8} \\
&f \left( \frac{9}{8} \right) = \frac{5}{9} \\
&f \left( \frac{9}{5} \right) = \frac{5}{7} \\
&f \left( \frac{9}{2} \right) = \frac{1}{2}
\end{align*}
\]

\[
A_5 = \frac{1}{5} \left( \frac{5}{6} \right) + \frac{1}{5} \left( \frac{5}{4} \right) + \frac{1}{5} \left( \frac{5}{8} \right) + \frac{1}{5} \left( \frac{5}{9} \right) + \frac{1}{5} \left( \frac{5}{7} \right)
\]

\[
= \frac{1}{6} + \frac{1}{4} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10}
\]

\[
A_5 \approx 0.6456
\]

Actual area = 0.6931

**Example 3:**

Suppose a particle moves with constant velocity \( 2 \text{ ft/sec} \)

\[
v(t) \quad \begin{array}{c}
\uparrow \\
2
\end{array}
\]

\[
A = \int v \, dt = bh
\]

\[
= (10 \text{ sec}) (2 \text{ ft/sec})
\]

\[
A = 20 \text{ ft}
\]

This is distance traveled on the interval \([0, 10]\)

- Question: what is the distance traveled for an arbitrary value of \( t \)?

\[
\begin{array}{c}
\uparrow \\
2
\end{array}
\quad \begin{array}{c}
\uparrow \\
t
\end{array}
\]

We want \( A(t) \)

\[
A(t) = (t \text{ sec}) (2 \text{ ft/sec})
\]

\[
A(t) = 2t \text{ ft}
\]

**Notice:** \( A(10) = 20 \text{ ft} \)

\[
A(5) = 10 \text{ ft}
\]

Recall: \( v(t) = v'(t) \)
Example 3: continued

Here we have \( v(t) = 2 t \) ft/sec.

- **Question:** What do we differentiate to get 2?

- **Answer:** \( 2t \)!

The process of finding \( s(t) \) from \( v(t) \) is called anti-differentiation.

**NOTICE:** If \( A(t) = 2t \), then \( A'(t) = 2 \).

- \( A(t) \) in this case is defined to be the anti-derivative function of \( v(t) \), which is \( s(t) \).

We should notice that the area under \( A(t) \) should represent \( v(t) \).

**NOTE:**

\[
\begin{align*}
A(t) &= \frac{1}{2} t \\
A &= \frac{1}{2} bh \\
A(\theta) &= \frac{1}{2} (8 \text{ sec})(1 \text{ ft/sec}) \\
&= 16 \text{ ft/sec}
\end{align*}
\]

The units tell us that this is velocity!

**Example 4:**

Find the area function of \( f(x) \) over the given interval.

a) \( f(x) = 3x + 1 \) on \([1, x]\)

\[
\begin{align*}
A_{\text{trap}} &= \frac{1}{2}(h_1 + h_2) b \\
A(x) &= \frac{1}{2}(4 + 3x + 1)(x-1) \\
h_1 &= 4 \\
h_2 &= 3x + 1 \\
b &= x-1
\end{align*}
\]

**NOTE:** \( A'(x) = 3x + 1 \)

b) \( f(x) = 2x - 3 \) on \([\frac{3}{2}, x]\)

\[
\begin{align*}
A_{\text{triangle}} &= \frac{1}{2} bh \\
A(x) &= \frac{1}{2} \left(x - \frac{3}{2}\right)(2x-3) \\
b &= x- \frac{3}{2}
\end{align*}
\]

\[
A(x) = \frac{1}{2} \left(2x^2 - 6x + \frac{9}{2}\right)
\]
Example 4: continues:

b) \[ A(x) = x^2 - 3x + \frac{4}{9} \]

NOTE: \[ A'(x) = 2x - 3 \]