SECTION: RECTILINEAR MOTION: REVISITED USING INTEGRATION

- Constant

\[ \int_a^t a \, dt = at + C_1 \]
\[ \therefore v(t) = at + C_2 \]

Notice: \( C_1 \) is initial velocity \( (v_0) \). \( v_0 = v(0) \)
\[ \therefore v(t) = at + v_0 \]

- To find \( s(t) \), we integrate \( v(t) \)
\[ \int v(t) \, dt \]
\[ = \int (at + v_0) \, dt \]
\[ = \frac{1}{2} at^2 + v_0 t + C_2 \]

- Theorem:
\[ s(t) = \frac{1}{2} at^2 + v_0 t + C_2 \]

Notice: \( s(0) = C_2 \rightarrow s(0) = \text{The initial position} \).

- Distanced traveled ≠ displacement

The particle returned to its initial position after the given time interval.

- Displacement

The net signed area is \( 0 \), this means the particle returned to its initial position.
DISTANCE TRAVELED
\[ \int_a^b |v(t)| \, dt \]

EXAMPLE 1: Find position function

a) \( v(t) = 3t^2 - 2t \quad s(0) = 1 \)

\[ s(t) = \int v(t) \, dt = \int (3t^2 - 2t) \, dt = \frac{3}{3} t^3 - \frac{2}{2} t^2 + C \]

\[ s(t) = t^3 - t^2 + C \]

Since \( s(0) = 1 \), we get \( 0^3 - 0^2 + C = 1 \), so \( C = 1 \).

\[ \therefore s(t) = t^3 - t^2 + 1 \]

EXAMPLE 2: A particle has a constant velocity of 25 cm/sec for 4 sec. Next, it experiences a negative acceleration of -10 cm/sec² on the time interval 4 < t < 12.

After 4 sec, \( a = -10 \) cm/sec²

\[ a(t) = \begin{cases} 0 & 0 < t < 4 \\ -10 & 4 < t < 12 \end{cases} \]

\[ v(t) = \int a(t) \, dt \]

For \( 0 < t < 4 \)
\[ v(0) = \int 0 \, dt = c = 25 \text{ cm/sec} \]

For \( 4 < t < 12 \)
\[ v(t) = \int (-10) \, dt = -10t + C \]

Since \( v_0 = 25 \text{ cm/sec} \) when \( 4 < t < 12 \) we have \( v(4) = 25 \text{ cm/sec} \).

\[ v(t) = -10t + C \]

\[ v(4) = -10(4) + C \]

\[ 25 = -40 + C \]

Solving for \( C \):
\[ C = 65 \]
Example 2: continues:

\[ v(t) = -10t + 65 \quad 9 < t < 12 \]

- When does \( v(t) = 0 \)?

\[ 0 = -10t + 65 \]
\[ 10t = 65 \]
\[ t = 6.5 \]

c) \[ s(t) = \int v(t) \, dt \]
\[ s(t) = \begin{cases} 
25t & 0 < t < 9 \\
-5t^2 + 65t - 80 & 9 < t < 12 
\end{cases} \]

\[ s(t) = \int (-10t + 65) \, dt \]
\[ s(t) = -5t^2 + 65t + C \quad 9 < t < 12 \]

**Note:** \( s(9) = 100 \)

\[ 100 = -5(9)^2 + 65(9) + C \]
\[ -80 = C \]
\[ \therefore s(t) = -5t^2 + 65t - 80 \]

d) What is the maximum positive position?

\[ s(6.5) = 131.25 \]

Example 3: constant acceleration

\[ 90 \text{ mi/hr} \quad 60 \text{ mi/hr} \]
\[ 200 \text{ ft} \]

\[ \frac{90}{30} \text{ mi/ft} \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left( \frac{1 \text{ hr}}{3600 \text{ sec}} \right) = 132 \text{ ft/sec} \]

\[ \frac{60}{30} \text{ mi/ft} \left( \frac{5280 \text{ ft}}{1 \text{ mi}} \right) \left( \frac{1 \text{ hr}}{3600 \text{ sec}} \right) = 88 \text{ ft/sec} \]
Example 3: continues

a) Find acceleration

"a" is constant

For constant acceleration,
\[ v(t) = \sqrt{a} \, dt = at + v_0 \]
\[ v(t) = at + 132 \]
\[ s(t) = \int v(t) \, dt \]
\[ = \int (at + 132) \, dt \]
\[ s(t) = \frac{a}{2} t^2 + 132t + s_0 \]

Note: We define \( s_0 = 0 \)

\[ \therefore s(t) = \frac{a}{2} t^2 + 132t \]

We can solve for \( t \) after the car travels 200 ft:
\[ 200 = \frac{a}{2} t^2 + 132t \]

We can also use the fact that after 200 ft of travel,
\[ v(t) = 88 \text{ ft/sec} \]
\[ 88 = at + 132 \]

Since \( v(t) = 88 = at + 132 \), we get \( 88 = at + 132 \), we get
\[ \frac{-44}{a} = t \]
\[ \frac{-44}{a} = t \text{ or } \frac{-44}{t} = a \]

We substitute our result into \( 200 = \frac{a}{2} t^2 + 132t \)
\[ 200 = \frac{(-44)}{2} t^2 + 132t \]
\[ 200 = -22t + 132t \]
\[ 200 = 110t \]
\[ t = \frac{200}{110} \]

To find acceleration, we will use \( v(t) \)
\[ v(t) = at + 132 \]
we know \( v \left( \frac{20}{11} \right) = 88 \)
\( 88 = a \left( \frac{20}{11} \right) + 132 \)
\( 44 = a \left( \frac{20}{11} \right) \)
\( -44 = a \left( \frac{11}{20} \right) \)
\( a = \frac{-242}{10} \)
\( a = -24.2 \text{ or } -\frac{121}{5} \)

- RECALL: constant acceleration
  \( a(t) = a \)
  \( v(t) = a(t) + v_0 \)
  \( s(t) = \frac{1}{2} at^2 + v_0 t + s_0 \)

- example 4: Find \( s(t) \) given the following information.
  \( a(t) = 2t^{-3} \)
  \( v(1) = 0 \)
  \( s(1) = 2 \)
  \( v(t) = \int a(t) \, dt \)
  \( = \int 2t^{-3} \, dt \)
  \( v(t) = -t^{-2} + C \)

To find \( C \) we use \( v(1) = 0 \)
\( v(t) = -t^{-2} + C \)
\( 0 = -1 + C \)
\( 1 = C \)
\( \therefore v(t) = -t^{-2} + 1 \)

\( s(t) = \int v(t) \, dt \)
\( = \int (-t^{-2} + 1) \, dt \)
\( s(t) = t^{-1} + t + C \)
\( s(1) = (1)^{-1} + (1) + C \)
\( 2 = 2 + C \)
\( 0 = C \)
Example 4: continues:

\[ s(t) = \frac{1}{t} + t \]