More with Fractions

Objective 1
Add and Subtract Fractions with Different Denominators

Remember: In order to add or subtract fractions the denominators must be the same.

Let's begin by working the problem \( \frac{2}{3} + \frac{1}{4} - \frac{5}{6} \).

In order to perform the indicated operations of addition and subtraction, we must rewrite each fraction as equivalent fractions having the same denominator.

We begin by first finding the **Least Common Denominator (LCD)** of all three fractions. The LCD can simply be thought of as the smallest number that all your denominators divide evenly into.

\[
\frac{2}{3} + \frac{1}{4} - \frac{5}{6}
\]

Here our denominators our 3, 4, and 6. The smallest number that 3, 4, and 6 divide evenly into is 12. Therefore 12 is the LCD.

Note: The LCD is never smaller than the largest denominator. In fact, it is always a multiple of the largest denominator.
Another method of finding the LCD is to find the **Least Common Multiple (LCM)** of the denominators. A simple way of doing this is to make a list of multiples of the denominators to find the lowest common multiple. This quantity will be the **LCD**.

For the problem \( \frac{2}{3} + \frac{1}{4} - \frac{5}{6} \), we will make list of multiples for 3, 4, and 6 starting with the largest denominator.

- 6: 6, 12, 18, 24, 30, 36, ...
- 4: 4, 8, 12, 16, 20, 24, 30, ...
- 3: 3, 6, 9, 12, 15, 18, 21, 24, ...

Notice that 12 is the lowest common multiple and therefore **12** is the LCD.

**Note:** When the denominators involve very large numbers, making a list of common multiples can be very time consuming. In these cases, using prime factorization to find the LCD may be a better approach. This method will be covered in a later section.
For the problem $\frac{2}{3} + \frac{1}{4} - \frac{5}{6}$ we have the LCD = 12. To rewrite each fraction as an equivalent fraction with a denominator of 12, we must multiply each fraction by an appropriate factor of 1.

\[
\frac{2}{3} + \frac{1}{4} - \frac{5}{6} = \\
\frac{2}{3}(-) + \frac{1}{4}(-) - \frac{5}{6}(-) = \\
\frac{8}{12} + \frac{3}{12} - \frac{10}{12} = \\
\frac{+ \quad -}{12} = \\
\frac{12}{12}
\]
Example 1: Perform the indicated operations.

a) \(-\frac{5}{6} + \frac{3}{10} - \frac{4}{5}\)

Multiples of Denominators
10: 10, 20, 30, 40, 50, 60, 70, ...
6: 6, 12, 18, 24, 30, 36, 42, ...
5: 5, 10, 15, 20, 25, 30, 35, ...

LCD = 30
\[-\frac{5}{6}(\frac{5}{5}) + \frac{3}{10}(\frac{3}{3}) - \frac{4}{5}(\frac{6}{6})\]
\[-\frac{25}{30} + \frac{9}{30} - \frac{40}{30}\]
\[-\frac{25 + 9 - 40}{30}\]
\[-\frac{64}{30}\]
\[-\frac{64}{30}\]
\[-\frac{28}{15}\]

b) \(-\frac{3}{8} + \left(-\frac{1}{4}\right)^2 - \frac{5}{32}\)

Multiples of Denominators.
32: 32, 64, 96, 128, ...
16: 16, 32, 48, 64, 80, 96, ...
8: 8, 16, 24, 32, 40, 48, 56, 64, ...

LCD = 32
\[-\frac{3}{8}(\frac{4}{4}) + \frac{1}{16}(\frac{2}{2}) - \frac{5}{32}\]
\[-\frac{12}{32} + \frac{2}{32} - \frac{5}{32}\]
\[-\frac{12 + 2 - 5}{32}\]
\[-\frac{15}{32}\]
Answer the following homework questions.

In Exercises 1 - 12, perform the indicated operations.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{3}{4} + \frac{2}{5} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{3}{4} + \frac{2}{5} - \frac{1}{10} )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{5}{9} - \left( -\frac{1}{6} \right) )</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{4}{3} + \frac{1}{7} ) (LCD = 3t)</td>
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<tr>
<td>5</td>
<td>( \frac{3}{12} - \left( -\frac{1}{2} \right)^3 )</td>
</tr>
<tr>
<td>6</td>
<td>( \frac{1}{8} - \left( -\frac{3}{4} \right)^2 )</td>
</tr>
<tr>
<td>7</td>
<td>( -\frac{3}{8} - \frac{2}{6} + \frac{1}{3} )</td>
</tr>
<tr>
<td>8</td>
<td>( 2 \frac{3}{5} + \frac{1}{2t} ) (LCD = 10h)</td>
</tr>
<tr>
<td>9</td>
<td>( \frac{2}{7} - \frac{2}{9} - \frac{2}{21} )</td>
</tr>
<tr>
<td>10</td>
<td>( \left( -\frac{2}{3} \right)^2 - \left( -\frac{2}{3} \right)^3 )</td>
</tr>
<tr>
<td>11</td>
<td>( \frac{6}{25} - \frac{2}{15} )</td>
</tr>
<tr>
<td>12</td>
<td>( \frac{3}{40} + \frac{5}{36} )</td>
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