The Distributive Property and Expressions

Objective 1

Understand how to use the Distributive Property to Clear Parenthesis

The Distributive Property

The Distributive Property states that multiplication can be distributed across addition and subtraction.

$$x(a+b) = ax + bx$$
$$a(x-y+z) = ax - ay + az$$
$$-a(x-y+z) = -ax + ay - az$$

Consider the expression $3(x+2)$. While the rules of **Order of Operations** state we must first work on the expression within the parenthesis, this cannot be done. The expression $x+2$ cannot be simplified since x and 2 are not like terms.

However, we can remove the parenthesis by distributing the 3 using multiplication to each term within the parenthesis.

$$3(x+2)$$
$$3 \cdot x + 3 \cdot 2$$
$$3x + 6$$

Suppose we need to remove the parenthesis from the expression $4(a+b-c)$. To do this, we will distribute the 4 using multiplication to each term within the parenthesis.

$$4(a+b-c)$$
$$4\cdot a+4\cdot b-4\cdot c$$
$$4a+4b-4c$$

When distributing a positive quantity across addition and subtraction within a parenthesis, the operations remain unchanged. But what happens when we distribute a negative quantity? You will notice that the operations will change. Can you explain why?

$$-4(a+b-c)$$

$$-4(a+b-c)$$
$$(-4)(a)+(-4)(b)-(-4)(c)$$
$$(-4a)+(-4b)-(-4c)$$
$$-4a-4b+4c$$

When distributing a negative quantity across addition, you end up adding a negative quantity. This is why addition changes to subtraction! Review how to add negative numbers.

When distributing a negative quantity across subtraction, you end up subtracting a negative quantity. This is why subtraction changes to addition! Review how to subtract negative numbers.

Most students **do not** write out all the steps when distributing a negative quantity across addition and subtraction. Most students choose to use "Kung Fu math". Try it on the following example.

Example 1: Apply the Distributive Property to remove the parenthesis. Use "Kung Fu math".

a) $-2(x-4)$ $\boxed{-2x+8}$

c) $-5(-8-t)$ $\boxed{40+5t}$

e) $-2(x-y+3)$ $\boxed{-2x+2y-6}$

b) $-5(a-2)$

d) $-10(-3-p)$

f) $-4(a-b+3)$

The expression $-(x-y+z)$ implies that a "-1" is being multiplied to the parenthesis. So writing down $-(x-y+z)$ is the same as writing down $-1(x-y+z)$. The process of distributing a negative is shown below.

$$-(x-y+z)$$
$$-x+y-z$$

In cases where we have an **addition** or **subtraction** symbol in front of a parenthesis, we must develop techniques to remove the parenthesis.

When there is an **addition** operation in front of a parenthesis, we can simply remove the parenthesis.

$$5+(x+y-z)=5+x+y-z$$

But if the first term in the parenthesis is negative, we must subtract its opposite! Once again, remember that adding a negative number results in subtracting its opposite!

$$5+(-x+y-z)=5-x+y-z$$

What happens when there is a **subtraction** operation in front of a parenthesis? Consider the following expression.

$$5-(x-y+z)$$

In this case we can treat the subtraction symbol as an addition of a "-1" and write the equivalent expression $5+(-1)(x-y+z)$. This approach is demonstrated below.

$$5-(x-y+z)$$

$$5+(-1)(x-y+z)$$ Rewrite the subtraction symbol as adding -1.

$$5+(-x+y-z)$$ Distribute the -1 into the parenthesis.

$$5-x+y-z$$ Remove the parenthesis.

When there is a quantity following the subtraction symbol, we use a similar approach. Note that you cannot do $10-8$ because the 8 is being multiplied to the parenthesis. You must perform multiplication before subtraction!

$$10-8(x-y+z)$$

$$10+(-8)(x-y+z)$$ Rewrite subtract 8 as adding -8.

$$10+(-8x+8y-8z)$$ Now distribute the -8 into the parenthesis.

$$10-8x+8y-8z$$ Remove the parenthesis.

Once you have practiced enough you will be able to correctly remove parenthesis without writing down all the steps.

Example 2: Simplify the expression by removing the parenthesis and combining like terms.

a) $3x + y - (x + 2y)$ b) $a + 2b + (-a + b)$

In some cases we have remove multiple sets of parenthesis before we can combine like terms. See the example below.

$$-3(2x + y) - 4(-3x - 2y)$$

$$-3(2x + y) - 4(-3x - 2y)$$
$$-6x - 3y + 12x + 8y$$

Here we use the Distributive Property to remove the parenthesis.

$$-6x - 3y + 12x + 8y$$
$$6x + 5y$$

Here we identify and combine like terms.

Example 3: Simplify the expression.

a) $3(a-2)$

e) $6(2x+1)-2$

b) $-(a-2)$

f) $-2(3y-3)+4y$

c) $3-(a-2)$

g) $2(x+1)+4(x-1)$

d) $3-6(a-2b)$

h) $-3(2x+5)-4(x-2)$

Objective 2

Find the Value of an Expression given the value of the Variable Term or Terms

We use **variables** to represent unknown quantities. In the expression $x+2$, the symbol x is the variable term. We cannot solve for x, as $x+2$ is not an equation, it is an expression. Equations have equal signs and expression do not!

We could find the value of the expression if we are given a number to represent the variable. In this case, we say we are evaluating the expression.

Example 4: Evaluate the following expressions given $x=12$.

a) $3x-8$ b) $-x\div 4$ c) $-x^2-44$

$3(12)-8$

$36-8$

$\boxed{28}$

Some expressions can have more than one variable. In these cases, you must be given the value of both variables to "find the value" or "evaluate" the expression.

Example 5: Evaluate the following expressions given that $x=3$ and $y=-2$.

a) $3x+2y$ b) x^2-y^2 c) $\frac{x^2}{y^2}$