### Objective 1

Understand how to use the Distributive Property to Clear Parenthesis

#### The Distributive Property

The Distributive Property states that multiplication can be distributed across addition and subtraction.

\[
x(a+b) = ax + bx \\
a(x-y+z) = ax - ay + az \\
-a(x-y+z) = -ax + ay - az
\]

Consider the expression \(3(x+2)\). While the rules of **Order of Operations** state we must first work on the expression within the parenthesis, this cannot be done. The expression \(x+2\) cannot be simplified since \(x\) and 2 are not like terms.

However, we can remove the parenthesis by distributing the 3 using multiplication to each term within the parenthesis.

\[
3(x+2) \\
3 \cdot x + 3 \cdot 2 \\
3x + 6
\]
Suppose we need to remove the parenthesis from the expression $4(a+b-c)$. To do this, we will distribute the 4 using multiplication to each term within the parenthesis.

\[ 4(a+b-c) \]
\[ 4 \cdot a + 4 \cdot b - 4 \cdot c \]
\[ 4a + 4b - 4c \]

When distributing a **positive** quantity across addition and subtraction within a parenthesis, the operations remain unchanged. But what happens when we distribute a **negative** quantity? You will notice that the operations will change. Can you explain why?

\[ -4(a+b-c) \]
\[ -(4)(a) + (-4)(b) - (-4)(c) \]
\[ -(4a) + (-4b) - (-4c) \]
\[ -4a - 4b + 4c \]
### Example 1: Apply the Distributive Property to remove the parenthesis. Use “Kung Fu math”.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2(x - 4)</td>
<td>-2x + 8</td>
</tr>
<tr>
<td>-5(-8 - t)</td>
<td>40 + 5t</td>
</tr>
<tr>
<td>e) -2(x - y + 3)</td>
<td>-2x + 2y - 6</td>
</tr>
<tr>
<td>b) -5(a - 2)</td>
<td></td>
</tr>
<tr>
<td>d) -10(-3 - p)</td>
<td></td>
</tr>
<tr>
<td>f) -4(a - b + 3)</td>
<td></td>
</tr>
</tbody>
</table>

Most students do not write out all the steps when distributing a negative quantity across addition and subtraction. Most students choose to use “Kung Fu math”. Try it on the following example.

When distributing a **negative quantity across addition**, you end up adding a negative quantity. This is why **addition changes to subtraction**! Review how to add negative numbers.

When distributing a **negative quantity across subtraction**, you end up subtracting a negative quantity. This is why **subtraction changes to addition**! Review how to subtract negative numbers.
The expression \(-(x-y+z)\) implies that a "\(-1\)" is being multiplied to the parenthesis. So writing down \(-(x-y+z)\) is the same as writing down \(-1(x-y+z)\). The process of distributing a negative is shown below.

\[
\begin{align*}
-(x-y+z) \\
-x+y-z
\end{align*}
\]

In cases where we have an addition or subtraction symbol in front of a parenthesis, we must develop techniques to remove the parenthesis.

When there is an addition operation in front of a parenthesis, we can simply remove the parenthesis.

\[5 + (x+y-z) = 5 + x + y - z\]

But if the first term in the parenthesis is negative, we must subtract its opposite! Once again, remember that adding a negative number results in subtracting its opposite!

\[5 + (-x+y-z) = 5 - x + y - z\]
What happens when there is a **subtraction** operation in front of a parenthesis? Consider the following expression.

\[ 5 - (x - y + z) \]

In this case we can treat the subtraction symbol as an addition of a "-1" and write the equivalent expression \[ 5 + (-1)(x - y + z) \]. This approach is demonstrated below.

\[ 5 - (x - y + z) \]
\[ 5 + (-1)(x - y + z) \]
\[ 5 + (-x + y - z) \]
\[ 5 - x + y - z \]

When there is a quantity following the subtraction symbol, we use a similar approach. Note that you cannot do \( 10 - 8 \) because the 8 is being multiplied to the parenthesis. You must perform multiplication before subtraction!

\[ 10 - 8(x - y + z) \]
\[ 10 + (-8)(x - y + z) \]
\[ 10 + (-8x + 8y - 8z) \]
\[ 10 - 8x + 8y - 8z \]
Once you have practiced enough you will be able to correctly remove parenthesis without writing down all the steps.

**Example 2:** Simplify the expression by removing the parenthesis and combining like terms.

\[
a) \; 3x + y - (x + 2y) \quad b) \; a + 2b + (-a + b)
\]

In some cases we have remove multiple sets of parenthesis before we can combine like terms. See the example below.

\[
-3(2x + y) - 4(-3x - 2y)
\]

Here we use the **Distributive Property** to remove the parenthesis.

\[
-3(2x + y) - 4(-3x - 2y) = -6x - 3y + 12x + 8y
\]

Here we identify and combine like terms.

\[
6x + 5y
\]
Example 3: Simplify the expression.

a) $3(a - 2)$  

b) $-(a - 2)$  

c) $3 - (a - 2)$  

d) $3 - 6(a - 2b)$

e) $6(2x + 1) - 2$

f) $-2(3y - 3) + 4y$

g) $2(x + 1) + 4(x - 1)$

h) $-3(2x + 5) - 4(x - 2)$
<table>
<thead>
<tr>
<th>Objective 2</th>
<th>Find the Value of an Expression given the value of the Variable Term or Terms</th>
</tr>
</thead>
</table>

We use **variables** to represent unknown quantities. In the expression \( x + 2 \), the symbol \( x \) is the variable term. We cannot solve for \( x \), as \( x + 2 \) is not an equation, it is an expression. Equations have equal signs and expression do not!

We could find the value of the expression if we are given a number to represent the variable. In this case, we say we are evaluating the expression.

**Example 4:** Evaluate the following expressions given \( x = 12 \).

a) \( 3x - 8 \)  
   \( 3(12) - 8 \)  
   \( 36 - 8 \)  
   \( 28 \)

b) \( -x \div 4 \)  
   \( -x^2 - 44 \)
Some expressions can have more than one variable. In these cases, you must be given the value of both variables to “find the value” or “evaluate” the expression.

**Example 5:** Evaluate the following expressions given that \(x=3\) and \(y=-2\).

a) \(3x + 2y\)  
b) \(x^2 - y^2\)  
c) \(\frac{x^2}{y^2}\)