

Solving Equations

Objective 1

Use Properties of Equality to Solve Equations

Recall that an expression, like $x+3$, can only be evaluated if given the value of the variable x . Expressions cannot be solved!

For example, if $x=5$ then the value of $x+3$ is 8.

But what about $x+3=8$? This is an equation because it contains an equal sign. In the case, we will be asked to solve the equation for the unknown value of x . By inspection the solution to the equation $x+3=8$ is $x=5$. This is because $5+3=8$.

Suppose we are asked to solve equation was $x-5=2$? By inspection the solution to the equation is $x=7$. This is because $7-5=2$.

So how do we solve equations algebraically? We can use "Properties of Equality" which state we can add, subtract, multiply, and divide a number to both sides of an equation without changing the solution.

The "Properties of Equality" will be demonstrated in the following four examples.

When solving an equation, our goal is to get the variable isolated on one side of the equation.

Example 1: Solve the equation $x - 5 = 2$ for x . Use the **Addition Property of Equality**.

In this example we must add 5 to both sides of the equation to isolate the variable.

$$x - 5 = 2$$

Vertical Method

$$\begin{array}{r} x - 5 = 2 \\ \quad \cancel{-5} \quad +5 \\ \hline x + 0 = 7 \\ \boxed{x = 7} \end{array}$$

Horizontal Method

$$\begin{array}{r} x - 5 + 5 = 2 + 5 \\ \cancel{x - 5} + \cancel{5} = 2 + 5 \\ x + 0 = 7 \\ \boxed{x = 7} \end{array}$$

Either method can be used to solve the equation for x . Remember to circle or box your final answer. Verify that your solution is correct by going back to the original equation and replacing the variable with your solution. Check that both sides of the equation are in fact equal.

Example 2: Solve the equation $x + 8 = 6$ for x .
 Use the **Subtraction Property of Equality**.

In this example we must subtract 8 to both sides of the equation to isolate the variable.

$$x + 8 = 6$$

Vertical Method

$$\begin{array}{r}
 x + 8 = 6 \\
 \underline{-8 \quad -8} \\
 x + 0 = -2 \\
 \boxed{x = -2}
 \end{array}$$

Horizontal Method

$$\begin{array}{r}
 x + 8 - 8 = 6 - 8 \\
 x + \cancel{8} - \cancel{8} = 6 - 8 \\
 x + 0 = -2 \\
 \boxed{x = -2}
 \end{array}$$

Example 3: Solve the equation $\frac{1}{3}x = 5$ for x .
 Use the **Multiplication Property of Equality**.

In this example we must multiply 3 to both sides of the equation to isolate the variable.

$$\begin{array}{r}
 \frac{1}{3}x = 5 \\
 3\left(\frac{1}{3}\right)(x) = 3(5) \quad \text{Multiply both sides by 3 to clear the fraction.} \\
 \overset{1}{\cancel{3}}\left(\frac{1}{\cancel{3}}\right)(x) = 3(5) \quad \text{Clear the fraction on the left side and multiply on the right.} \\
 \overset{1}{1}(x) = 15 \quad \text{Multiply 1(x) to get x.} \\
 \boxed{x = 15}
 \end{array}$$

The equation $\frac{1}{3}x = 5$ is equivalent to the equation $\frac{x}{3} = 5$ and can be solved using the same technique.

$$\frac{x}{3} = 5$$

$$3\left(\frac{x}{3}\right) = 3(5) \quad \text{Multiply both sides by 3 to clear the fraction.}$$

$$\cancel{3}\left(\frac{x}{\cancel{3}}\right) = 3(5) \quad \text{Clear the fraction on the left side and multiply on the right.}$$

$$x = 15$$

Example 4: Solve the equation $12x = 5$ for x .
 Use the **Division Property of Equality**.

In this example we must divide 12 to both sides of the equation to isolate the variable.

$$12x = 5$$

$$\frac{12x}{12} = \frac{5}{12}$$

Divide both sides by 12 to get a 1 in front of the x .

$$\cancel{12}x = \frac{5}{12}$$

Reduce the fraction to $1x$.

$$1x = \frac{5}{12}$$

$$x = \frac{5}{12}$$

Write $1x$ as x .

Consider the equation below.

$$\frac{x}{4} + \frac{2}{3} = \frac{5}{6}$$

Notice that we have fractions on both sides of the equation. To clear the fractions or "Kung Fu" them, we can multiply both sides of the equation by the LCD of all three fractions!

Before we move forward, let's review how to clear fraction.

$12\left(\frac{x}{4}\right)$	$12\left(\frac{2}{3}\right)$	$12\left(\frac{5}{6}\right)$
$\overset{3}{\cancel{12}}\left(\frac{x}{\cancel{4}}\right)$ 1	$\overset{4}{\cancel{12}}\left(\frac{2}{\cancel{3}}\right)$ 1	$\overset{2}{\cancel{12}}\left(\frac{5}{\cancel{6}}\right)$ 1
$3(x)$	$4(2)$	$2(5)$
$3x$	8	10

These calculations will be used in the following example.

Example 5: Solve the equation.

$$\frac{x}{4} + \frac{2}{3} = \frac{5}{6} \quad \text{LCD} = 12$$

$$12 \left(\frac{x}{4} + \frac{2}{3} \right) = 12 \left(\frac{5}{6} \right) \quad \text{Multiply both sides by 12.}$$

$$12 \left(\frac{x}{4} \right) + 12 \left(\frac{2}{3} \right) = 12 \left(\frac{5}{6} \right) \quad \text{Apply the distributive property to the left side of the equation.}$$

$$3x + 8 = 10 \quad \text{clear the fractions.}$$

$$3x + 8 = 10$$

$$\underline{\quad - 8 \quad - 8}$$

$$3x + 0 = 2$$

$$3x = 2$$

Subtract 8 from both sides of the equation to isolate the **variable term** on the left side of the equation.

$$\frac{3x}{3} = \frac{2}{3}$$

Divide both sides of the equation by 3 to isolate the **variable** on the left side.

$$\boxed{x = \frac{2}{3}}$$

Answer the following homework questions.

In Exercises 1 - 15, solve each equation for the unknown.

1) $x + 4 = 12$

6) $-7w + 4 + 8w = 9 - 12$

2) $p - 8 = 13$

7) $-6 - 4s - 11 + 8s = 6 - 18$

3) $3t - 5 = 7$

8) $8 - (y + 4) - 8 + 2y = -18$

4) $2m + 4 = -16$

9) $5b - 10 + 3(-2 - b) = 12$

5) $3 + 4r = 12$

10) $-(a + 4) - 2(-2a + 4) = 12$

11) $\frac{1}{4}x = \frac{2}{3}$

12) $k + \frac{1}{2} = \frac{3}{4}$

13) $-\frac{3}{5}c = \frac{8}{15}$

14) $\frac{3}{4}t = \frac{5}{2} - \frac{1}{6}$

15) $\frac{1}{4}d - \frac{2}{3}d + \frac{1}{5} = 2 - \frac{2}{3}$