More Equations

**Objective 1** Solve Equations with Variable Terms on both Sides of the Equation

Some equations like $5x - 3 = 4 + 8x$ have variable terms on both sides of the equations. As always, we must isolate the variable term to one side. It does not matter which side. Below the equation is solved in two different ways.

$$5x - 3 = 4 + 8x$$

$$\begin{align*}
5x &- 3 = 4 + 8x \\
-8x &- 8x \\
-3x & = 4 \\
-3 & = 4 + 3x \\
-3 + 3 & = 4 + 3x + 3 \\
3 & = 7 \\
-3x & = 7 \\
-3 & = 3x \\
\frac{-3}{3} & = \frac{7}{3} \\
& = \frac{-7}{3} \\
x & = -\frac{7}{3}
\end{align*}$$

$$\begin{align*}
5x &- 3 = 4 + 8x \\
-5x &- 5x \\
-3x & = 4 \\
-3 & = 4 + 3x \\
-4 & = 4 + 3x - 4 \\
-4 & = 3x \\
-7 & = 3x \\
\frac{-7}{3} & = \frac{3x}{3} \\
& = \frac{-7}{3} \\
x & = -\frac{7}{3}
\end{align*}$$

-or-

$$\begin{align*}
5x &- 3 = 4 + 8x \\
-8x &- 8x \\
-3x & = 4 \\
-3 & = 4 + 3x \\
-3 + 3 & = 4 + 3x + 3 \\
3 & = 7 \\
-3x & = 7 \\
-3 & = 3x \\
\frac{-3}{3} & = \frac{7}{3} \\
& = \frac{-7}{3} \\
x & = -\frac{7}{3}
\end{align*}$$
Answer the following homework questions.

In Exercises 1 - 10, solve each equation for the unknown.

1) \( 6 = x - 13 \)
2) \( -4 = 3r + 8 \)
3) \( p = 2p + 15 \)
4) \( -2k + 3 = -15 - 4k \)
5) \( x + 2 = 8 - 5x \)
6) \( -7h + 4 + 8h = 8 - 6h \)
7) \( -w - 11 = 6 + w \)
8) \( -1 + y = 7 - 2(y + 4) \)
9) \( -2(-b - 5) = 12 + 3(-2 - b) \)
10) \( 5(4 + 2e) = 12e - 2(-2e + 4) \)

**Objective 2: Solve Equations using a Reciprocal**

Some equations like \( \frac{5}{6}x = \frac{20}{3} \) can be solved very quickly by multiplying both sides of the equation by the reciprocal of the fraction in front of the variable. Note that multiplying reciprocals together will always result in 1.

\[
\frac{5}{6}x = \frac{20}{3}
\]

Multiply both sides by the reciprocal of \( \frac{5}{6} \), which is \( \frac{6}{5} \).

\[
\frac{6}{5} \left( \frac{5}{6} \right) x = \frac{6}{5} \left( \frac{20}{3} \right)
\]

Clear the fraction on the left side and reduce the fraction on the right side.

\[
\frac{1}{5} \left( \frac{5}{1} \right) x = \frac{1}{5} \left( \frac{20}{1} \right)
\]

\[
x = 8
\]
using a reciprocal will not work efficiently when given an equation like \( \frac{5}{6}x = \frac{20}{3} - \frac{1}{4} \). You would have to distribute on the right side of the equation. You will likely end up with a fraction on the right side of the equation.

\[
\frac{5}{6}x = \frac{20}{3} - \frac{1}{4}
\]

\[
\frac{6}{5} \left( \frac{5}{6} \right) x = \frac{6}{5} \left( \frac{20}{3} - \frac{1}{4} \right)
\]

\[
\frac{6}{5} \left( \frac{5}{6} \right) x = \frac{6}{5} \left( \frac{20}{3} \right) - \frac{6}{5} \left( \frac{1}{4} \right)
\]

\[
x = 8 - \frac{3}{10}
\]

\[
x = \frac{80}{10} - \frac{3}{10} \quad \text{LCD=10}
\]

\[
x = \frac{77}{10} \quad \text{or} \quad 7 \frac{7}{10}
\]

Try working the problem \( \frac{5}{6}x = \frac{20}{3} - \frac{1}{4} \) using the clearing fraction technique with an LCD of 12. Using “Kung Fu math” you will immediately get \( 10x = 80 - 3 \). We can quickly solve this to get the solution!
Answer the following homework questions.

In Exercises 11 - 20, solve each equation for the unknown. Begin each problem by multiplying both sides of the equation with the reciprocal of the quantity in front of the variable.

11) \( \frac{5}{8} p = \frac{15}{4} \)  
12) \( \frac{3}{20} t = \frac{9}{10} \)  
13) \( -\frac{3}{11} y = 6 \)  
14) \( \frac{4}{5} x = 7 \)  
15) \( -\frac{3}{5} r = 10 \)  
16) \( 2m = \frac{12}{5} \)  
17) \( -6w = -\frac{8}{3} \)  
18) \( 21b = -\frac{63}{2} \)  
19) \( 8a = -\frac{32}{5} \)  
20) \( -\frac{8}{3} e = -8 \)