

**§1-3****PROPORTION AND PERCENT****Definition**

A **ratio** is an ordered pair of numbers written  $\frac{x}{y}$  where  $y \neq 0$ .

**Theorem****Equality of Ratios**

$\frac{a}{b} = \frac{c}{d}$  if and only if  $ad = bc$ , where  $a, b, c$  and  $d$  are real numbers and  $b \neq 0, d \neq 0$ .

**Definition**

A **proportion** is a statement that two ratios are equal.

**Example 1**

Sam uses three tablespoons of instant cocoa to make two servings of hot cocoa. How many tablespoons of instant cocoa does Sam need if he is to make five servings of hot cocoa?

**Solution**

Sam uses a ratio of 3 tablespoons of mix to 2 servings of hot water. Since Sam needs the same ratio of tablespoons of mix to 5 servings of hot water we get the following proportion.

$$\frac{3}{2} = \frac{x}{5}$$

where  $x$  is the number of tablespoons of mix required. Solving this equation for  $x$ :

$$2x = 15$$

$$x = \frac{15}{2}$$

$$x = 7\frac{1}{2} \text{ tablespoons}$$

**Definition**

**Percent** means parts per hundred. Thus  $n\%$  represents the ratio  $\frac{n}{100}$ .

**Example 2**

What is 30% of 150?

**Solution**

$$(30\%)(150) = (0.30)(150) = 45$$

**Example 3**

15 is what percentage of 180?

**Solution**

$$15 \div 180 = 0.08\bar{3} = 8.\bar{3}\% = 8\frac{1}{3}\%$$

**Example 4**

Suppose apples cost \$1.15 per pound. If oranges cost 20% more than apples, how much will three pounds of oranges cost?

**Solution**

$$\text{We first calculate } 20\% \text{ of } \$1.15: (20\%)(\$1.15) = (0.2)(\$1.15) = \$0.23.$$

Therefore each pound of oranges cost 23 cents more than each pound of apples.

This implies that the cost of 1 pound of oranges is  $\$1.15 + .23 = \$1.38$ .

We conclude 3 pounds of oranges costs,  $3(\$1.38) = \$4.14$ .

Find the ratio of each pair of quantities.

1. 8 feet to 4 inches
2.  $2\frac{1}{2}$  gallons to 3 quarts
3. 40 seconds to  $\frac{1}{3}$  minute
4. \$1.65 to 5¢
5. 1 hour and 15 minutes to 5 hours

In each of the following proportions, find the value of  $x$ .

6.  $\frac{3}{5} = \frac{15}{x}$
7.  $\frac{x}{49} = \frac{9}{21}$
8.  $\frac{3}{x} = \frac{2}{15}$
9.  $\frac{1.5}{4} = \frac{8.4}{x}$
10.  $\frac{1}{27} = \frac{x}{3}$
11.  $\frac{2x}{5} = \frac{4}{3}$
12.  $\frac{9}{2} = \frac{3x}{14}$
13.  $\frac{33}{x} = \frac{11}{5}$
14.  $\frac{2}{x} = \frac{b}{3}$
15.  $\frac{9}{x} = \frac{x}{16}$
16.  $\frac{r}{s} = \frac{c}{x}$
17.  $\frac{b}{x} = \frac{3}{b^2}$

Solve each proportion problem.

18. Candy bars sell at a rate of 3 for 42¢. What will 10 candy bars cost?
19. A wheelbarrow can carry 53 lbs in 7 loads. How many trips are needed to haul 140 lbs?
20. If three painters could paint four apartments in a day, then how many apartments could five painters paint in a day?
21. 55 miles per hour is approximately 88 kilometers per hour. If a car travels at 90 miles per hour, then approximately how many kilometers per hour is it traveling?
22. The scale on a map is  $\frac{3}{4}$  inches to 6 miles. How far apart are two cities if the map shows them as 4 inches apart?

Solve each percentage problem.

23. What is 12% of 1300?
24. 238 is what percentage of 1400?
25. A jacket regularly costs \$104.20. If it is on sale at 20% off then how much does it cost?
26. If a realtor earns a 5% commission on every house that she sells, then how much will she earn on the sale of a \$123,000 house?
27. At Albert Einstein Junior High School there are 123 students who participate in sports. If this represents 15% of the student body, then how many students attend Albert Einstein JHS?

**§1-3****PROBLEM SOLUTIONS**

1.  $\frac{24}{1}$       2.  $\frac{10}{3}$       3.  $\frac{2}{1}$       4.  $\frac{33}{1}$       5.  $\frac{1}{4}$
6. 25      7. 21      8.  $22\frac{1}{2}$       9. 22.4      10.  $\frac{1}{9}$
11.  $3\frac{1}{3}$       12. 21      13. 15      14.  $\frac{6}{b}$       15.  $\pm 12$
16.  $\frac{cs}{r}$       17.  $\frac{b^3}{3}$       18. \$1.40      19. 19 trips      20.  $6\frac{2}{3}$
21. 144 kph      22. 32 miles      23. 156      24. 17%      25. \$83.36
26. \$6150      27. 820 students