

§6-6**RIGHT TRIANGLE TRIGONOMETRY**

In this section we consider the trigonometry of right triangles. The trigonometric functions of concern are the *sine*, *cosine* and *tangent* functions.

Definition

For a right triangle, the **sine** of an angle θ is the ratio of the length of the leg opposite θ to the length of the hypotenuse: $\sin \theta = \frac{\text{opposite leg}}{\text{hypotenuse}}$.

Definition

For a right triangle, the **cosine** of an angle θ is the ratio of the length of the leg adjacent θ to the length of the hypotenuse: $\cos \theta = \frac{\text{adjacent leg}}{\text{hypotenuse}}$.

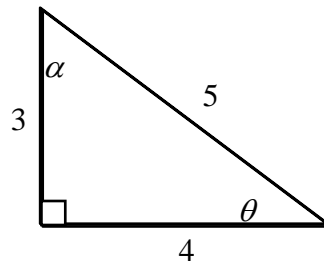
Definition

For a right triangle, the **tangent** of an angle θ is the ratio of the length of the leg opposite θ to the length of the leg adjacent θ : $\tan \theta = \frac{\text{opposite leg}}{\text{adjacent leg}}$.

The mnemonic SOH CAH TOA is often used to recall the relationship of the sides of the right triangle to the trigonometric functions.

Example 1

Find the sine, cosine and tangent of θ and α in the right triangle shown below.

**Solution**

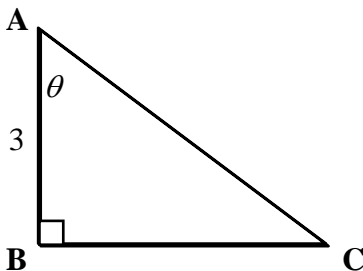
For θ the opposite leg has length 3 and the adjacent leg has length 4. Therefore...

$$\sin \theta = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{3}{5} \quad \cos \theta = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{4}{5} \quad \tan \theta = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{3}{4}$$

For α the opposite leg has length 4 and the adjacent leg has length 3. Therefore...

$$\sin \alpha = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{4}{5} \quad \cos \alpha = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{3}{5} \quad \tan \alpha = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{4}{3}$$

Example 2 If $\cos \theta = x$ for the triangle below, then find the length of **AC**.



Solution

$$\cos \theta = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$
$$x = \frac{3}{\mathbf{AC}}$$
$$\mathbf{AC}x = 3$$
$$\mathbf{AC} = \frac{3}{x}$$

Identity For any angle θ , $\sin^2 \theta + \cos^2 \theta = 1$.

Example 3 If $\sin \theta = \frac{1}{2}$ then find the cosine of θ .

Solution

$$\sin^2 \theta + \cos^2 \theta = 1$$
$$\cos^2 \theta = 1 - \sin^2 \theta$$
$$\cos^2 \theta = 1 - \left(\frac{1}{2}\right)^2$$
$$\cos^2 \theta = \frac{3}{4}$$
$$\cos \theta = \pm \frac{\sqrt{3}}{2}$$

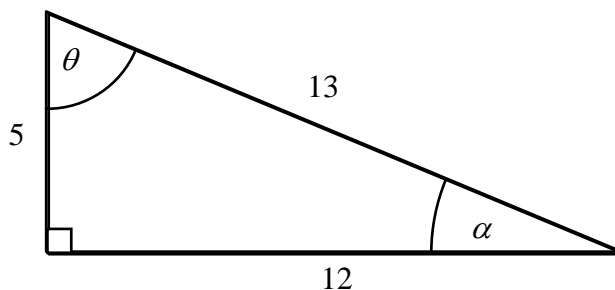
Identity For any angle θ , $\tan \theta = \frac{\sin \theta}{\cos \theta}$.

Example 4 If $\cos \theta = \frac{5}{13}$ and $\tan \theta = \frac{12}{5}$ then find the sine of θ .

Solution

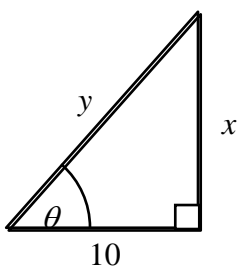
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$
$$\tan \theta \cdot \cos \theta = \sin \theta$$
$$\frac{12}{5} \cdot \frac{5}{13} = \sin \theta$$
$$\frac{12}{13} = \sin \theta$$

Find each quantity for the right triangle below.



- | | | |
|------------------|------------------|------------------|
| 1. $\sin \alpha$ | 2. $\cos \alpha$ | 3. $\tan \alpha$ |
| 4. $\sin \theta$ | 5. $\cos \theta$ | 6. $\tan \theta$ |

Find the lengths of the missing sides of the right triangle below.



$$\cos \theta = \frac{2}{3}$$

$$\tan \theta = \frac{\sqrt{5}}{2}$$

- | | |
|---------------|---------------|
| 7. Find x . | 8. Find y . |
|---------------|---------------|

Use the given quantities to calculate the missing quantity.

9. If $\sin \theta = \frac{1}{2}$ and $\cos \theta = \frac{\sqrt{3}}{2}$ then find $\tan \theta$.

10. If $\sin \theta = \frac{\sqrt{2}}{2}$ and $\tan \theta = 1$ then find $\cos \theta$.

11. If $\cos \theta = \frac{1}{3}$ and $\tan \theta = 2\sqrt{2}$ then find $\sin \theta$.

12. If $\cos \theta = \frac{\sqrt{5}}{3}$ and $\sin \theta > 0$ then find $\sin \theta$.

1. $\frac{5}{13}$

2. $\frac{12}{13}$

3. $\frac{5}{12}$

4. $\frac{12}{13}$

5. $\frac{5}{13}$

6. $\frac{12}{5}$

7. 15

8. $5\sqrt{5}$

9. $\frac{\sqrt{3}}{3}$

10. $\frac{\sqrt{2}}{2}$

11. $\frac{2\sqrt{2}}{3}$

12. $\frac{2}{3}$