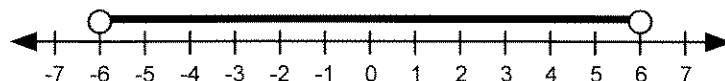


§4-3**ABSOLUTE VALUE INEQUALITIES****Theorem**

If $|u| < c$ where u is a variable expression and c is non-negative, then $-c < u < c$.

Example 1 Solve the inequality, $|x| < 6$.

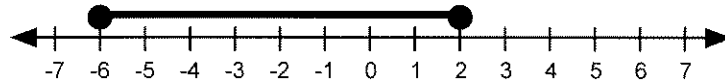
Solution If $|x| < 6$ then $-6 < x < 6$.



Example 2 Solve the inequality, $|x + 2| \leq 4$.

Solution If $|x + 2| \leq 4$ then $-4 \leq x + 2 \leq 4$.

$$\begin{aligned} -4 &\leq x + 2 \leq 4 \\ -4 - 2 &\leq x + 2 - 2 \leq 4 - 2 \\ -6 &\leq x \leq 2 \end{aligned}$$



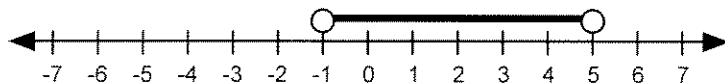
Example 3 Solve the inequality, $-3|x - 2| + 1 > -8$.

Solution

$$\begin{aligned} -3|x - 2| + 1 &> -8 \\ -3|x - 2| + 1 - 1 &> -8 - 1 \\ -3|x - 2| &> -9 \\ \frac{-3|x - 2|}{-3} &< \frac{-9}{-3} \\ |x - 2| &< 3 \end{aligned}$$

If $|x - 2| < 3$ then $-3 < x - 2 < 3$.

$$\begin{aligned} -3 &< x - 2 < 3 \\ -3 + 2 &< x - 2 + 2 < 3 + 2 \\ -1 &< x < 5 \end{aligned}$$

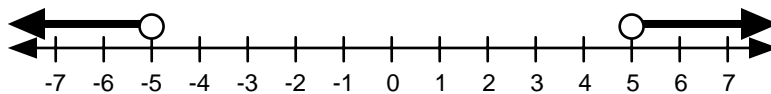


Theorem

If $|u| > c$ where u is a variable expression and c is non-negative, then $u > c$ or $u < -c$.

Example 4 Solve the inequality, $|x| > 5$.

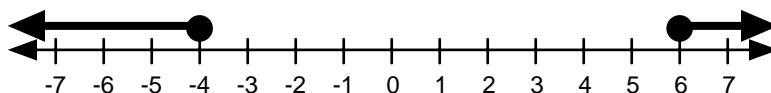
Solution If $|x| > 5$ then $x > 5$ or $x < -5$.



Example 5 Solve the inequality, $|x-1| \geq 5$.

Solution If $|x-1| \geq 5$ then $x-1 \geq 5$ or $x-1 \leq -5$.

$$\begin{array}{rcl} x-1 & \geq & 5 \\ x-1+1 & \geq & 5+1 \\ x & \geq & 6 \end{array} \quad \text{or} \quad \begin{array}{rcl} x-1 & \leq & -5 \\ x-1+1 & \leq & -5+1 \\ x & \leq & -4 \end{array}$$



Example 6 Solve the inequality, $-2|x-1|+10 < 2$.

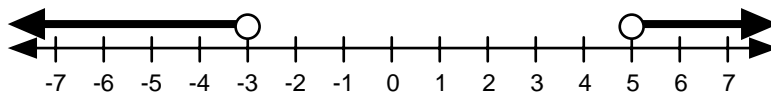
Solution

$$\begin{aligned} -2|x-1|+10 &< 2 \\ -2|x-1|+10-10 &< 2-10 \\ -2|x-1| &< -8 \end{aligned}$$

$$\frac{-2|x-1|}{-2} > \frac{-8}{-2}$$

$$|x-1| > 4$$

$$\begin{array}{rcl} \text{If } |x-1| > 4 \text{ then } & x-1 > 4 & \text{or} & x-1 < -4. \\ & x-1 > 4 & & x-1 < -4 \\ & x-1+1 > 4+1 & & x-1+1 < -4+1 \\ & x > 5 & & x < -3 \end{array}$$



Solve each absolute value inequality, and graph the solution on a number line.

1. $|x| > 4$

2. $|x| + 2 < 5$

3. $|3v| \geq \frac{5}{2}$

4. $|x| + 3 \leq 5$

5. $\left| \frac{y}{3} \right| < 5$

6. $|y - 5| > 9$

7. $3|t| \leq 12$

8. $|3t| \geq 12$

9. $|p| - \frac{1}{2} > \frac{1}{4}$

10. $\left| 4 - \frac{b}{2} \right| < 3$

11. $|x - 4| \geq 2$

12. $|5x + 1| \leq 10$

13. $\frac{1}{3}|w| < 4$

14. $4|x| > 20$

15. $|-2x + 1| \leq 8$

16. $|3a - a| \geq 3$

17. $|4x| > 20$

18. $\frac{1}{2}|2x + 1| < 15$

19. $\frac{1}{3}|x + 2| \geq \frac{2}{3}$

20. $-|x - 3| + 5 \leq 1$

21. $2 - 3|1 - x| < -1$

22. $|7z + 1| > 2$

23. $4|3a - 1| - 4 \leq -3$

24. $\left| \frac{2}{3}x + 4 \right| \geq 16$

25. $|1 + 3q| > \frac{1}{5}$

26. $|3x + 1| + 6 < 10$

27. $|3x - 1| \geq 4$

28. $5|4x - 3| + 7 \leq 52$

29. $\frac{1}{3}|3a - 7| + 8 < 10$

30. $|2x + 9| > 11$

31. $\left| \frac{1}{2}t \right| \leq 4$

32. $5|4x - 3| + 7 \geq 52$

33. $|4x - 7| - 5 > -5$

34. $|3x + 2| - 4 < 4$

35. $|3n + 4| \geq \frac{1}{2}$

36. $5|4x - 7| - 5 \leq -5$

37. $|3x - 2| + 1 < -1$

38. $-3|2 - x| + 5 > -13$

39. $\left| \frac{3x - 2}{5} \right| \leq 1$

40. $|-6y - 3| \geq 4$

41. $\frac{1}{2}|2 - x| > 1$

42. $-11 < -1 - 3|2 - x|$

43. $|y - 9| \geq 5$

44. $\left| \frac{4}{5}r + 9 \right| \leq \frac{1}{3}$

45. $3|h - 14| - 7 > 2$

1. $x > 4$ or $x < -4$
2. $-3 < x < 3$
3. $v \geq \frac{5}{6}$ or $v \leq -\frac{5}{6}$
4. $-2 \leq x \leq 2$
5. $-15 < y < 15$
6. $y > 14$ or $y < -4$
7. $-4 \leq t \leq 4$
8. $t \geq 4$ or $t \leq -4$
9. $p > \frac{3}{4}$ or $p < -\frac{3}{4}$
10. $2 < b < 14$
11. $x \geq 6$ or $x \leq 2$
12. $-\frac{11}{5} \leq x \leq \frac{9}{5}$
13. $-12 < w < 12$
14. $x > 5$ or $x < -5$
15. $-\frac{7}{2} \leq x \leq \frac{9}{2}$
16. $a \geq \frac{3}{2}$ or $a \leq -\frac{3}{2}$
17. $x > 5$ or $x < -5$
18. $-\frac{31}{2} < x < \frac{29}{2}$
19. $x \geq 0$ or $x \leq -4$
20. $x \geq 7$ or $x \leq -1$
21. $x < 0$ or $x > 2$
22. $z > \frac{1}{7}$ or $z < -\frac{3}{7}$
23. $\frac{1}{4} \leq a \leq \frac{5}{12}$
24. $x \geq 18$ or $x \leq -30$
25. $q > -\frac{4}{15}$ or $q < -\frac{2}{5}$
26. $-\frac{5}{3} < x < 1$
27. $x \geq \frac{5}{3}$ or $x \leq -1$
28. $-\frac{3}{2} \leq x \leq 3$
29. $\frac{1}{3} < a < \frac{13}{3}$
30. $x > 1$ or $x < -10$
31. $-8 \leq t \leq 8$
32. $-\frac{3}{2} \leq x \leq 3$
33. $x > \frac{7}{4}$ or $x < \frac{7}{4}$
34. $-2 < x < \frac{10}{3}$
35. $n \geq -\frac{7}{6}$ or $n \leq -\frac{3}{2}$
36. $x = \frac{7}{4}$
37. no solution
38. $-4 < x < 8$
39. $-1 \leq x \leq \frac{7}{3}$
40. $y \leq -\frac{7}{6}$ or $y \geq \frac{1}{6}$
41. $x < 0$ or $x > 4$
42. $-\frac{4}{3} < x < \frac{10}{3}$
43. $y \geq 14$ or $y \leq 4$
44. $-\frac{35}{3} < r < -\frac{65}{6}$
45. $n > 17$ or $n < 11$