

§7-4**STATISTICS****Definition**

Statistics is the science of conducting studies to collect, organize, summarize, analyze, and draw conclusions from data.

When working with large collections of data, the concept of “average” is very useful. There are many types of average, but those most commonly used are called the *mean*, the *median*, and the *mode*.

Definition

The **mean** is sometimes referred to as the *arithmetic average* of the data. It is the sum of the values of the items in the data set divided by the number of items in the data set.

$$\text{The mean} = \frac{a_1 + a_2 + \dots + a_n}{n} \text{ where } a_1, a_2, \dots, a_n \text{ are the values of } n \text{ items.}$$

Example 1

Find the mean of the following data: 1, 4, 2, 6, 2.

Solution

$$\text{mean} = \frac{1 + 4 + 2 + 6 + 2}{5} = \frac{15}{5} = 3$$

Definition

The **median** is the middle value when the data is ordered from smallest value to largest value. If there are an even number of values then the median is the average of the two middle values.

Example 2

Find the median of the following data: 1, 4, 2, 6, 2.

Solution

Ordering the data from smallest value to largest value gives us 1 2 2 4 6.
The middle number is the second 2 so 2 is the median.

Definition

The **mode** is the value that most frequently occurs in the data set. It is possible to have no mode or more than one mode. A set of data with two modes is called a **bimodal** distribution.

Example 3

Find the mode of the following data: 1, 4, 2, 6, 2.

Solution

Since the number 2 appears twice and each other number appears only once, 2 is the mode.

Example 4 A class earns the following scores on a pop-quiz: 0, 52, 52, 60, 72, 82, 85, 85, 99, 100. Find the mean the median and mode(s) for these class scores.

Solution The mean
The scores add up to 687. Since there are 10 scores the mean is given by:

$$\text{mean} = \frac{687}{10} = 68.7$$

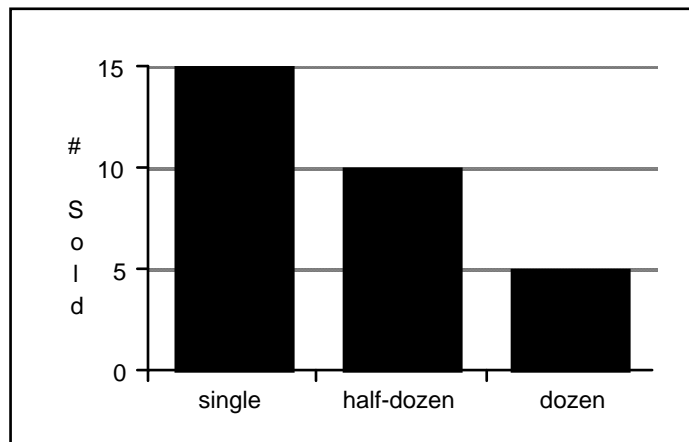
The median
Ordering the scores from smallest to largest gives us:

0 52 52 60 72 82 85 85 99 100

Since there are an even number of scores, there are two middle numbers: 72 and 82.
The median is the average of these two numbers: $\frac{72 + 82}{2} = 77$

The mode
The most frequently occurring scores are 52 and 85, both of which occur twice. Thus 52 and 85 are both modes.

Example 5 A bakery sells donuts individually, in packages of 6 and in packages of 12. The sales for one day are shown below. If there were 50 customers, then what is the average number of donuts purchased by each customer?



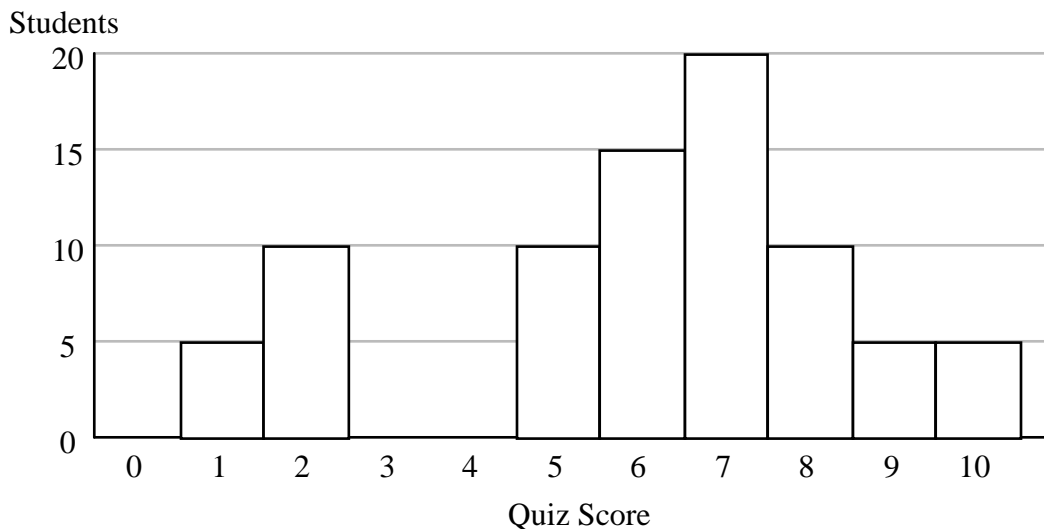
Solution There were 15 individual donuts sold: $15 \cdot 1 = 15$
There were 10 packages of a half-dozen sold: $10 \cdot 6 = 60$
There were 5 packages of a dozen sold: $5 \cdot 12 = 60$

Thus a total of 135 donuts were sold to 50 customers so the mean is $\frac{135}{50} = 2.7$.

§7-4**PROBLEM SET**

Find the Mean, Median and Mode(s) of each set of data.

1. { 4, 7, 2, 4, 3 }
2. { 7, 1, 3, 9, 5, 6, 3, 7 }
3. { -2.5, 1.25, -4, 2.75, 3.5 }
4. { 6, 9, 2, 5, 14, 6, 8 }
5. { 38, 42, 35, 17, 24, 38, 21 }
6. { 5, 12, 15, 5, 9, 12, 17, 5 }
7. { 42, 60, 36, 52, 48, 36, 42 }
8. { 127, 108, 120, 116, 127, 118 }
9. { 96, 94, 88, 89, 88 }
10. { 70, 70, 90, 100, 75, 60, 95 }
11. { 14.0, 13.4, 13.2, 14.0, 13.9 }
12. { 1.4, 32.4, 64.4, 18, 32.4, 25, 10 }
13. { 10.5, 30.6, 98.6, 101, 48.8, 46.8 }
14. { 126, 147, 140, 126, 136, 131, 118 }
15. { 32, 28, 19, 24, 28, 19, 26, 27, 30, 28, 25 }
16. { 38, 40, 20, 21, 30, 33, 24, 20, 37, 40 }
17. { 20, 14, 52, 40, 39, 18, 14, 15, 0, 38 }
18. { 11,500; 101,000; 51,000; 75,000; 8,500 }
19. { 9.16, 8.44, 7.60, 4.35, 0.97, 4.35, 8.44 }
20. The following histogram represents the quiz scores for a group of students. Find the mean, median and mode of the set of data.



1. mean = 4; median = 4; mode = 4
2. mean = 5.125; median = 5.5; mode = 7 and 3
3. mean = 0.2; median = 1.25; mode = none
4. mean \approx 7.14; median = 6; mode = 6
5. mean \approx 30.71; median = 35; mode = 38
6. mean = 10; median = 10.5; mode = 5
7. mean \approx 45.14; median = 42; mode = 42 and 36
8. mean \approx 119.3; median = 119; mode = 127
9. mean = 91; median = 89; mode = 88
10. mean = 80; median = 75; mode = 70
11. mean = 13.7; median = 13.9; mode = 14.0
12. mean \approx 26.23; median = 25; mode = 32.4
13. mean = 56.05; median = 47.8; mode = none
14. mean = 132; median = 131; mode = 126
15. mean = 26; median = 27; mode = 28
16. mean = 30.3; median = 31.5; mode = 40 and 20
17. mean = 25; median = 19; mode = 14
18. mean = 49,400; median = 51,000; mode = none
19. mean \approx 6.19; median = 7.60; mode = 8.44 and 4.35
20. mean = 6; median = 6.5; mode = 7