

§2-4**RADICAL EXPRESSIONS****Definition**

The **n th root** of a number a is a number that when raised to the n th power, produces the number a . It is written as $\sqrt[n]{a}$, where n is the **index**, $\sqrt{\quad}$ is the **radical** and a is the **radicand**.

For example, $\sqrt{9} = 3$ since $3^2 = 9$ and $\sqrt{x^6} = x^3$ since $(x^3)^2 = x^6$.
Similarly, $\sqrt[4]{16} = 2$ since $2^4 = 16$ and $\sqrt[3]{x^{12}} = x^4$ since $(x^4)^3 = x^{12}$.

Definition

Rational exponents are exponents of the form $\frac{m}{n}$, where m is the power of the number and n represents the n th root of the number. Thus $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ where $a \geq 0$.

For example, $27^{1/3} = \sqrt[3]{27} = 3$ and $8^{2/3} = (\sqrt[3]{8})^2 = (2)^2 = 4$

Example 1 Write the expression $x^{2/3}$ in radical form.

Solution The expression $x^{2/3}$ can be written as $(x^{1/3})^2$ or $(x^2)^{1/3}$.
Therefore, $x^{2/3}$ can be written in radical form as $(\sqrt[3]{x})^2$ or $\sqrt[3]{x^2}$.

Example 2 Simplify the expression $\sqrt{18} - \sqrt{8}$.

Solution $\sqrt{18} - \sqrt{8} = \sqrt{9 \cdot 2} - \sqrt{4 \cdot 2} = 3\sqrt{2} - 2\sqrt{2} = (3 - 2)\sqrt{2} = \sqrt{2}$

Example 3 Simplify the expression $\sqrt[3]{x^7 y^5 z^6} - \sqrt[3]{x^{10} y^8 z^3}$.

Solution

$$\begin{aligned} \sqrt[3]{x^7 y^5 z^6} - \sqrt[3]{x^{10} y^8 z^3} &= \sqrt[3]{x^6 \cdot x \cdot y^3 \cdot y^2 \cdot z^6} - \sqrt[3]{x^9 \cdot x \cdot y^6 \cdot y^2 \cdot z^3} \\ &= (x^2 y z^2) \sqrt[3]{x y^2} - (x^3 y^2 z) \sqrt[3]{x y^2} \\ &= (x^2 y z^2 - x^3 y^2 z) \sqrt[3]{x y^2} \\ &= x^2 y z (z - xy) \sqrt[3]{x y^2} \end{aligned}$$

Example 4 Multiply $(\sqrt{2} - \sqrt{3})(\sqrt{8} - \sqrt{4})$.

Solution

$$\begin{aligned} (\sqrt{2} - \sqrt{3})(\sqrt{8} - \sqrt{4}) &= \sqrt{2} \cdot \sqrt{8} - \sqrt{2} \cdot \sqrt{4} - \sqrt{3} \cdot \sqrt{8} + \sqrt{3} \cdot \sqrt{4} \\ &= \sqrt{16} - \sqrt{8} - \sqrt{24} + \sqrt{12} \\ &= 4 - \sqrt{4 \cdot 2} - \sqrt{4 \cdot 6} + \sqrt{4 \cdot 3} \\ &= 4 - 2\sqrt{2} - 2\sqrt{6} + 2\sqrt{3} \end{aligned}$$

Write each exponential expression as a radical expression.

1. $5^{1/2}$ 2. $(2x)^{1/3}$ 3. $(xy^3)^{3/4}$ 4. $-5x^{2/3}$ 5. $(a^2b^4)^{1/5}$

Write each radical expression as a exponential expression.

6. $\sqrt{11}$ 7. $\sqrt[5]{a^3}$ 8. $-\sqrt{2y}$ 9. $4x\sqrt{x}$ 10. $2r^4\sqrt[4]{2s^3}$

Simplify each radical expression.

11. $\sqrt{64}$ 12. $\sqrt[3]{64}$ 13. $\sqrt[3]{-1}$ 14. $\sqrt[3]{-27}$ 15. $\sqrt{\frac{4}{9}}$

16. $\sqrt{x^{12}}$ 17. $-\sqrt{a^{12}b^4}$ 18. $\sqrt[3]{8x^6y^9}$ 19. $\sqrt[4]{16r^8}$ 20. $\sqrt[5]{-x^5y^{10}}$

21. $\sqrt[3]{-27a^9}$ 22. $\sqrt[5]{243x^{15}y^{25}}$ 23. $\sqrt{\frac{25x^2}{y^4}}$ 24. $\sqrt[3]{\frac{27a^{12}}{b^{15}}}$

25. $\sqrt{(x+y)^2}$ 26. $\sqrt[3]{(x+y)^6}$ 27. $3\sqrt{5} + 2\sqrt{5}$ 28. $3\sqrt{5} - 2\sqrt{5}$

29. $2\sqrt{12} - 3\sqrt{3}$ 30. $a\sqrt{7} - b\sqrt{7}$ 31. $3 \cdot \sqrt[3]{16} - \sqrt[3]{2}$ 32. $\sqrt{x^3} - 2x\sqrt{x}$

33. $\sqrt{x} + \sqrt{x}$ 34. $\sqrt{x} \cdot \sqrt{x}$ 35. $3\sqrt{2} \cdot \sqrt{3}$ 36. $\sqrt{x+y} \cdot \sqrt{x+y}$

37. $\sqrt{6x} \cdot \sqrt{12x}$ 38. $\frac{2}{\sqrt{2}}$ 39. $\frac{x}{\sqrt{x}}$ 40. $\frac{2x}{\sqrt{8x^3}}$

41. $\sqrt[3]{\sqrt{64}}$ 42. $\sqrt[3]{\sqrt{x}}$ 43. $\sqrt[3]{\sqrt{x^{12}}}$ 44. $\sqrt{\sqrt[3]{4ab^2}}$

45. $\frac{5\sqrt{x}}{y^2} \cdot \frac{\sqrt{25y}}{\sqrt{x^3}}$ 46. $\frac{\sqrt{3y}}{4\sqrt{4x}} \cdot \frac{x}{y\sqrt{y}}$ 47. $\frac{5\sqrt{x}}{y^2} \div \frac{\sqrt{25y}}{\sqrt{x^3}}$ 48. $\frac{\sqrt{3y}}{4\sqrt{4x}} \div \frac{x}{y\sqrt{y}}$

49. $\frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}}$ 50. $\frac{4}{\sqrt{2}} - \frac{3}{\sqrt{2}}$ 51. $\frac{\sqrt{27}}{\sqrt{12}} + \frac{2}{\sqrt[3]{27}}$

52. $\frac{2}{\sqrt{12}} - \frac{3}{\sqrt{3}}$ 53. $\frac{2}{\sqrt{12}} + \frac{3}{\sqrt{3}}$ 54. $\frac{\sqrt{27}}{\sqrt{12}} - \frac{2}{\sqrt[3]{27}}$

55. $\sqrt{2}(\sqrt{10} + \sqrt{5})$ 56. $\sqrt{3}(\sqrt{8} - \sqrt{6})$ 57. $\frac{2}{\sqrt{3} - \sqrt{2}}$

58. $\frac{2}{\sqrt{3} + \sqrt{2}}$ 59. $\frac{3}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{3} - \sqrt{2}}$ 60. $\frac{2}{\sqrt{2} + \sqrt{5}} - \frac{3}{\sqrt{2} - \sqrt{5}}$

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|-----|-------------------------|-----|----------------------------------|-----|-----------------------------|-----|--------------------------|-----|---------------------------|
| 1. | $\sqrt{5}$ | 2. | $\sqrt[3]{2x}$ | 3. | $y^2 \cdot \sqrt[4]{x^3y}$ | 4. | $-5 \cdot \sqrt[3]{x^2}$ | 5. | $\sqrt[5]{a^2b^4}$ |
| 6. | $11^{1/2}$ | 7. | $a^{3/5}$ | 8. | $-(2y)^{1/2}$ | 9. | $4x^{3/2}$ | 10. | $(2^{7/4})(r)(s^{3/4})$ |
| 11. | 8 | 12. | 4 | 13. | -1 | 14. | -3 | 15. | $\frac{2}{3}$ |
| 16. | x^6 | 17. | $-a^6b^2$ | 18. | $2x^2y^3$ | 19. | $2r^2$ | 20. | $-xy^2$ |
| 21. | $-3a^3$ | 22. | $3x^3y^5$ | 23. | $\frac{5x}{y^2}$ | 24. | $\frac{3a^4}{b^5}$ | 25. | $x + y$ |
| 26. | $(x + y)^2$ | 27. | $5\sqrt{5}$ | 28. | $\sqrt{5}$ | 29. | $\sqrt{3}$ | 30. | $(a - b)\sqrt{7}$ |
| 31. | $5 \cdot \sqrt[3]{2}$ | 32. | $-x\sqrt{x}$ | 33. | $2\sqrt{x}$ | 34. | x | 35. | $3\sqrt{6}$ |
| 36. | $x + y$ | 37. | $2x\sqrt{18}$ | 38. | $\sqrt{2}$ | 39. | \sqrt{x} | 40. | $\frac{\sqrt{2x}}{2x}$ |
| 41. | 2 | 42. | $\sqrt[6]{x}$ | 43. | x^2 | 44. | $\sqrt[24]{ab^2}$ | 45. | $\frac{25\sqrt{y}}{xy^2}$ |
| 46. | $\frac{\sqrt{3x}}{8y}$ | 47. | $\frac{x^2\sqrt{y}}{y^3}$ | 48. | $\frac{y^2\sqrt{3x}}{8x^2}$ | 49. | $\frac{7\sqrt{2}}{2}$ | 50. | $\frac{\sqrt{2}}{2}$ |
| 51. | $\frac{13}{6}$ | 52. | $-\frac{2\sqrt{3}}{3}$ | 53. | $\frac{4\sqrt{3}}{3}$ | 54. | $\frac{5}{6}$ | 55. | $2\sqrt{5} + \sqrt{10}$ |
| 56. | $2\sqrt{6} - 3\sqrt{2}$ | 57. | $2\sqrt{3} + 2\sqrt{2}$ | 58. | $2\sqrt{3} - 2\sqrt{2}$ | | | | |
| 59. | $4\sqrt{3} - 2\sqrt{2}$ | 60. | $\frac{\sqrt{2} + 5\sqrt{5}}{3}$ | | | | | | |