Disclaimer

You should use this practice exam to assess your speed and to improve your ability to correctly identify different problem types. The questions on this practice exam are taken from exams given in previous semesters, but they may not be representative of the questions that will appear on this semester's exam. You should also invest time re-reading the relevant parts of your textbook, reviewing your notes, and practicing homework problems.
1. Approximately 17% of children born in the U.S. have blue eyes.\(^1\) A random sample of thirty children born in the U.S. is selected.

a. Find the mean, variance, and standard deviation of the number of children who have blue eyes.

\[ S = \text{child has blue eyes} \quad p = 0.17 \]
\[ F = \text{child does not have blue eyes} \quad q = 0.83 \]
\[ n = 30 \]

\[ \mu = np = 30(0.17) = 5.1 \]

\[ \sigma^2 = npq = 30(0.17)(0.83) = 4.233 \approx 4.2 \]

\[ \sigma = \sqrt{4.233} = 2.057 \ldots \approx 2.1 \]

b. Find the probability that fewer than three of the thirty children have blue eyes.

\[ P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) \]
\[ = \begin{array}{c}
\binom{30}{0}(0.17)^0(0.83)^{30} + \\
\binom{30}{1}(0.17)^1(0.83)^{29} + \\
\binom{30}{2}(0.17)^2(0.83)^{28}
\end{array} \]
\[ = 1(0.83)^{30} + 30(0.17)(0.83)^{29} + 435(0.17)^2(0.83)^{28} \]
\[ = 0.003735 + 0.022953 + 0.068167 \]
\[ = 0.094855 \]
\[ \approx 0.0949 \]

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\(^1\) Douglas Belkin, "Blue eyes are increasingly rare in America," International Herald Tribune 10/18/2006
2. For each question below, find the probability using the standard normal distribution. (36 points)

a. $P(z < -1.56)$

$P(z < -1.56) = 0.0594$

b. $P(z > 2.26)$

$P(z > 2.26) = 1 - 0.9881$

$= 0.0119$

c. $P(z > -0.78)$

$P(z > -0.78) = 1 - 0.2177$

$= 0.7823$

d. $P(z < 0.54)$

$P(z < 0.54) = 0.7054$

e. $P(z < 2.3)$

$P(z < 2.3) = 0.9893$

f. $P(z > -1.4)$

$P(z > -1.4) = 1 - 0.0808$

$= 0.9192$

g. $P(1.15 < z < 1.88)$

$P(1.15 < z < 1.88) = 0.9699 - 0.8749$

$= 0.095$

h. $P(-0.42 < z < 2.09)$

$P(-0.42 < z < 2.09) = 0.9817 - 0.3372$

$= 0.6445$
3. In a game of chance, five $1 bills, four $5 bills, three $10 bills, two $20 bills, and one $100 bill are placed in a box. A person is charged $10 to select one bill at random. (34 points)

a. Construct the probability distribution for this game in table form. 
   Express all probabilities as decimals or reduced fractions.

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>5</th>
<th>0</th>
<th>10</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>1/3</td>
<td>4/15</td>
<td>1/5</td>
<td>2/15</td>
<td>1/15</td>
</tr>
</tbody>
</table>

b. Find the mean, variance, and standard deviation for this game.

\[ \mu = \sum X \cdot P(X) \]
\[ = (-9)\left(\frac{1}{3}\right) + (-5)\left(\frac{4}{15}\right) + (0)\left(\frac{1}{5}\right) + (10)\left(\frac{2}{15}\right) + (90)\left(\frac{1}{15}\right) \]
\[ = -3 + \frac{-4}{3} + 0 + \frac{4}{3} + 6 \]
\[ = $3 \]

\[ \sigma^2 = \sum X^2 \cdot P(X) - \mu^2 \]
\[ = (-9)^2\left(\frac{1}{3}\right) + (-5)^2\left(\frac{4}{15}\right) + (0)^2\left(\frac{1}{5}\right) + (10)^2\left(\frac{2}{15}\right) + (90)^2\left(\frac{1}{15}\right) - (3)^2 \]
\[ = 27 + \frac{20}{3} + 0 + \frac{40}{3} + 540 - 9 \]
\[ = 578 \]
\[ \sigma = \sqrt{\sigma^2} = \sqrt{578} \approx $24.04 \]

c. Is this game fair? Briefly explain your answer.

No, because \( \mu \neq 0 \).