

You will not receive full credit if you do not clearly show work as demonstrated in class. Show all work in the space provided on this exam. Circle your answers.

1. The probability that a customer will buy a television set and buy an extended warranty is 0.03. If the probability that a customer will purchase a television set is 0.12, find the probability that a customer will buy an extended warranty given that he is buying a TV. (8 points)

Since we are trying to find a conditional probability, we use the equation $P(B | A) = \frac{P(B \text{ and } A)}{P(A)}$.

$$\text{So } P(\text{warranty} | \text{TV}) = \frac{P(\text{warranty and TV})}{P(\text{TV})} = \frac{0.03}{0.12} = 0.25$$

2. Find the probability that if 4 different-sized washers are arranged in a row, they will be arranged in order of size. (8 points)

There are two ways to arrange the washers in order of size: smallest to largest and largest to smallest. There are $4!$ ways to arrange four distinct washers, so...

$$P(\text{arranged in order}) = \frac{2}{4!} = \frac{2}{24} = \frac{1}{12}$$

3. How many different ID cards can be made if there are 5 digits on a card and no digit can be used more than once on a particular card? (8 points)

There are ten possible digits and five are selected for each ID card. Since order matters...

$${}_{10}P_5 = \frac{10!}{(10-5)!} = \frac{10!}{5!} = \frac{3,628,800}{120} = 30,240$$

4. A single card is selected at random from a regular deck of 52 cards. (22 points)

a. Find $P(\text{the card is a face card}^1)$

$$P(\text{face}) = \frac{12}{52} = \frac{3}{13}$$

b. Find $P(\text{the card is not a face card})$

$$P(\text{not a face}) = 1 - P(\text{face}) = 1 - \frac{3}{13} = \frac{10}{13}$$

c. Find $P(\text{the card is a heart})$

$$P(\text{heart}) = \frac{13}{52} = \frac{1}{4}$$

d. Find $P(\text{the card is a heart and the card is a face card})$

$$P(\text{heart and face}) = \frac{3}{52}$$

e. Find $P(\text{the card is a heart or the card is a face card})$

$$\begin{aligned} P(\text{heart or face}) &= P(\text{heart}) + P(\text{face}) - P(\text{heart and face}) \\ &= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} \\ &= \frac{22}{52} \\ &= \frac{11}{26} \end{aligned}$$

¹ Note: Face cards are Kings, Queens, and Jacks.

5. The probability that a person owns a microwave oven is 0.75, that a person owns a compact disk player is 0.35, and that a person owns both a microwave and a CD player is 0.16. Find the probability that a person owns either a microwave or a CD player. (8 points)

$$\begin{aligned} P(\text{microwave or CD}) &= P(\text{microwave}) + P(\text{CD}) - P(\text{microwave and CD}) \\ &= 0.75 + 0.35 - 0.16 \\ &= 0.94 \end{aligned}$$

6. If a die is rolled eleven times, find the probability of getting at least one '3'. (8 points)

$$\begin{aligned} P(\text{at least one 3}) &= 1 - P(\text{no 3's}) \\ &= 1 - [P(\text{no 3 on one roll})]^{11} \\ &= 1 - \left(\frac{5}{6}\right)^{11} \\ &\approx 1 - 0.1346 \\ &= 0.8654 \end{aligned}$$

7. Complete each formula below for n distinct items taken r at a time. (8 points)

a. ${}_n P_r = \frac{n!}{(n-r)!}$

b. ${}_n C_r = \frac{n!}{(n-r)!(r!)}$

8. A person randomly selects three marbles (without replacement) from a jar containing 6 red marbles, 2 white marbles, and 7 blue marbles. Find the probability of selecting: (20 points)

- a. all blue marbles

$$P(3 \text{ blue}) = \frac{{}_7C_3}{{}_{15}C_3} = \frac{35}{455} = \frac{1}{13}$$

- b. two red marbles and one blue marble

$$P(2 \text{ red, 1 blue}) = \frac{{}_6C_2 \cdot {}_7C_1}{{}_{15}C_3} = \frac{15 \cdot 7}{455} = \frac{105}{455} = \frac{3}{13}$$

- c. both white marbles and one of a different color

$$P(2 \text{ white, 1 other}) = \frac{{}_2C_2 \cdot {}_{13}C_1}{{}_{15}C_3} = \frac{1 \cdot 13}{455} = \frac{13}{455} = \frac{1}{35}$$

- d. one marble of each color

$$P(1 \text{ red, 1 white, 1 blue}) = \frac{{}_6C_1 \cdot {}_2C_1 \cdot {}_7C_1}{{}_{15}C_3} = \frac{6 \cdot 2 \cdot 7}{455} = \frac{84}{455} = \frac{12}{65}$$

9. A manufacturer makes two types of mp3 player: an expensive model that accounts for 25% of their unit sales and a cheaper model that accounts for the other 75% of their unit sales. Twenty percent of the expensive model are colored pink, and ten percent of the cheaper model are colored pink. If an mp3 player made by this company is randomly selected, what is the probability that it is pink? (10 points)

A pink mp3 player can either be the expensive model or the cheaper one, so...

$$\begin{aligned}
 P(\text{pink}) &= P(\text{expensive and pink}) + P(\text{cheap and pink}) \\
 &= P(\text{expensive}) \cdot P(\text{pink} \mid \text{expensive}) + P(\text{cheap}) \cdot P(\text{pink} \mid \text{cheap}) \\
 &= (0.25)(0.2) + (0.75)(0.1) \\
 &= 0.05 + 0.075 \\
 &= 0.125
 \end{aligned}$$

Formulas

$$P(A \text{ or } B) = P(A) + P(B)$$

$$P(E) + P(\bar{E}) = 1$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(\bar{E}) = 1 - P(E)$$

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

$$P(\bar{E}) = 1 - P(E)$$

$$P(A \text{ and } B) = P(A) \cdot P(B)$$