Math 10: Exam 3 **Solutions**

You will not receive full credit if you do not clearly show work as demonstrated in class. Show all work in the space provided on this exam. Circle your answers.

1. A particular gumball machine is filled with gumballs of several colors which are dispensed randomly. Eight percent of the gumballs are purple. A child purchases forty-five gumballs. (30 points)

a. Find the mean, variance and standard deviation for the number of purple gumballs purchased.

S = purple $\quad p = 0.08$

F = not purple $\quad q = 0.92$

$n = 45$

$$\mu = np = 45(0.08) = 3.6$$

$$\sigma^2 = npq = 45(0.08)(0.92) = 3.312 \approx 3.3$$

$$\sigma = \sqrt{npq} = \sqrt{3.312} = 1.81989 \approx 1.8$$

b. Find the probability that at least three purchased gumballs are purple.

$$\begin{aligned}
P(\text{at least 3 purple}) &= P(X \geq 3) \\
&= 1 - P(X < 3) \\
&= 1 - \left[P(X=0) + P(X=1) + P(X=2)\right] \\
&= 1 - \left[\frac{45!}{(45!)(0!)}p^0q^{45} + \frac{45!}{(44!)(1!)}p^1q^{44} + \frac{45!}{(43!)(2!)}p^2q^{43}\right] \\
&= 1 - \left[1(0.08)^0(0.92)^{45} + 45(0.08)^1(0.92)^{44} + 990(0.08)^2(0.92)^{43}\right] \\
&= 1 - \left[0.023467 + 0.091826 + 0.175668\right] \\
&= 1 - 0.290961 \\
&= 0.709039 \\
&\approx 0.709
\end{aligned}$$

2. The world's smallest mammal is the Kitti's hog-nosed bat, with a mean weight of 150 centigrams and a standard deviation of 25 centigrams. Assuming that the weights are normally distributed, find the probability of randomly selecting a bat that weighs… (36 points)

a. more than 109 centigrams.

$$
\begin{aligned}
P(X > 109) &= P\left(\frac{X-\mu}{\sigma} > \frac{109-150}{25}\right) \\
&= P(z > -1.64) \\
&= 1 - 0.0505 \\
&= 0.9495
\end{aligned}
$$

b. more than 203 centigrams.

$$
\begin{aligned}
P(X > 203) &= P\left(\frac{X-\mu}{\sigma} > \frac{203-150}{25}\right) \\
&= P(Z > 2.12) \\
&= 1 - 0.9830 \\
&= 0.0170
\end{aligned}
$$

c. between 166 and 212.5 centigrams.

$$
\begin{aligned}
P(166 < X < 212.5) &= P\left(\frac{166-150}{25} < \frac{X-\mu}{\sigma} < \frac{212.5-150}{25}\right) \\
&= P(0.64 < z < 2.5) \\
&= 0.9938 - 0.7389 \\
&= 0.2549
\end{aligned}
$$

d. between 117.5 and 208 centigrams.

$$
\begin{aligned}
P(117.5 < X < 208) &= P\left(\frac{117.5-150}{25} < \frac{X-\mu}{\sigma} < \frac{208-150}{25}\right) \\
&= P(-1.3 < z < 2.32) \\
&= 0.9898 - 0.0968 \\
&= 0.8930
\end{aligned}
$$

3. In a game of chance, five \$1 bills, four \$5 bills, three \$10 bills, two \$20 bills, and one \$100 bill are placed in a box. A person is charged \$10 to select one bill at random. (34 points)

a. Construct the probability distribution for this game in table form.
Express all probabilities as decimals or reduced fractions.

X	-9	-5	0	10	90
$P(X)$	1/3	4/15	1/5	2/15	1/15

b. Find the mean, variance, and standard deviation for this game.

$$\begin{aligned}\mu &= \sum X \cdot P(X) \\ &= (-9)\left(\frac{1}{3}\right) + (-5)\left(\frac{4}{15}\right) + (0)\left(\frac{1}{5}\right) + (10)\left(\frac{2}{15}\right) + (90)\left(\frac{1}{15}\right) \\ &= -3 + -\frac{4}{3} + 0 + \frac{4}{3} + 6 \\ &= \$3\end{aligned}$$

$$\begin{aligned}\sigma^2 &= \sum X^2 \cdot P(X) - \mu^2 \\ &= (-9)^2\left(\frac{1}{3}\right) + (-5)^2\left(\frac{4}{15}\right) + (0)^2\left(\frac{1}{5}\right) + (10)^2\left(\frac{2}{15}\right) + (90)^2\left(\frac{1}{15}\right) - (3)^2 \\ &= 27 + \frac{20}{3} + 0 + \frac{40}{3} + 540 - 9 \\ &= 578\end{aligned}$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{578} \approx \$24.04$$

c. Is this game *fair*? Briefly explain your answer.

No, because $\mu \neq 0$.

Formulas $\sigma^2 = \sum X^2 \cdot P(X) - \mu^2$ $z = \frac{X - \mu}{\sigma}$ $\sigma^2 = npq$

$X = \mu + z\sigma$ $P(X) = \frac{n!}{(n-X)!X!} p^X q^{n-X}$ $\mu = np$ $\mu = \sum X \cdot P(X)$