Math 10: Exam 4 **Solutions**

You will not receive full credit if you do not clearly show work as demonstrated in class. Show all work in the space provided on this exam. Circle your answers.

1. An educator desires to estimate the true proportion of high school students who study at least one hour each school night. He wants to be 98% confident that his estimate is within 3% of the true proportion. If no estimate of the sample proportion is available, how large should the sample be? (12 points)

With no estimate available we assume that $\hat{p} = 0.5$ and $\hat{q} = 0.5$.

$$\begin{cases} \hat{p} = 0.5 \\ \hat{q} - 0.5 \\ E = 0.03 \\ z_{\alpha/2} = 2.326 \end{cases}$$

$$\begin{aligned} n &= \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2 \\ &= (0.5)(0.5)\left(\frac{2.326}{0.03}\right)^2 \\ &= 1502.854 \end{aligned}$$

So n must be at least 1503.

2. The average teacher's salary in North Dakota is \$29,863. Assume that the salaries are normally distributed with a standard deviation of \$5100. What is the probability that the mean for a sample of 80 teachers' salaries is greater than \$30,000? (16 points)

$$\begin{cases} \mu = 29{,}863 \\ \sigma = 5100 \\ n = 80 \end{cases}$$

$$\begin{aligned} P(\bar{X} > 30{,}000) &= P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > z = \frac{30{,}000 - 29{,}863}{5100/\sqrt{80}}\right) \\ &= P(z > 0.24) \\ &= 1 - 0.5948 \\ &= 0.4052 \end{aligned}$$

3. One group of 10 men and one group of 35 women had their heart rates monitored while being subjected to a stress situation. (40 points)

a. For the men, the mean number of heartbeats per minute was 126, and the standard deviation of the sample was 4. Find the 95% confidence interval of the true mean.

$$\begin{cases} \bar{X} = 126 \\ s = 4 \\ n = 10 \\ d.f. = 9 \\ t_{\alpha/2} = 2.262 \end{cases}$$

$$\begin{aligned} E &= t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \\ &= 2.262\left(\frac{4}{\sqrt{10}}\right) \\ &= 2.86 \\ &\approx 3 \end{aligned}$$

$$\begin{aligned} \bar{X} - E < \mu < \bar{X} + E \\ 126 - 3 < \mu < 126 + 3 \\ 123 < \mu < 129 \end{aligned}$$

b. For the women, the mean number of heartbeats per minute was 115. Assume that the population standard deviation is known to be 3. Find the 99% confidence interval of the true mean.

$$\begin{cases} \bar{X} = 115 \\ \sigma = 3 \\ n = 35 \\ z_{\alpha/2} = 2.576 \end{cases}$$

$$\begin{aligned} E &= z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \\ &= 2.576\left(\frac{3}{\sqrt{35}}\right) \\ &= 1.306 \\ &\approx 1 \end{aligned}$$

$$\begin{aligned} \bar{X} - E < \mu < \bar{X} + E \\ 115 - 1 < \mu < 115 + 1 \\ 114 < \mu < 116 \end{aligned}$$

4. The probability of winning on a certain slot machine is 12%. A person plays this slot machine one-hundred times. Use the normal approximation to the binomial distribution to find the probability that this person will win at least 18 times. (32 points)

Success = the person wins a game $p = 0.12$

Failure = the person loses a game $q = 0.88$

$n = 100$

$\mu = np = (100)(0.12) = 12$

$\sigma = \sqrt{npq} = \sqrt{(100)(0.12)(0.88)} = 3.25$

$P(X \geq 18) = ?$

Since $np \geq 5$ and $nq \geq 5$ we can use the normal approximation to the binomial distribution.

We want to include 18 in our approximation so we change the endpoint of the interval to 17.5.

$$P(X > 17.5) = P\left(\frac{X-\mu}{\sigma} > \frac{17.5-12}{3.25}\right)$$
$$= P(z > 1.69)$$
$$= 1 - 0.9545$$
$$= 0.0455$$

Formulas

$\hat{p} - E < p < \hat{p} + E$	$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$	$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$	$\hat{p} = \frac{X}{n}$	$\hat{q} = 1 - \hat{p} = \frac{n-X}{n}$
$\bar{X} - E < \mu < \bar{X} + E$	$E = z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$	$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$	$\mu = np$	$z = \frac{X-\mu}{\sigma}$
$\bar{X} - E < \mu < \bar{X} + E$	$E = t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$	$d.f. = n - 1$	$\sigma = \sqrt{npq}$	$z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$