Deriving the Shortcut Formula for the Sample Variance

Steps	Result
Given.	$s^2 = \dfrac{\Sigma(X - \bar{X})^2}{n-1}$
Square the binomial in the numerator.	$s^2 = \dfrac{\Sigma(X^2 - 2\bar{X} \cdot X + \bar{X}^2)}{n-1}$
Split up the sum.	$s^2 = \dfrac{(\Sigma X^2) - \Sigma(2\bar{X} \cdot X) + \Sigma(\bar{X}^2)}{n-1}$
Factor out the constants 2 and $\bar{X}$.	$s^2 = \dfrac{(\Sigma X^2) - 2\bar{X} \cdot (\Sigma X) + \bar{X}^2 \cdot (\Sigma 1)}{n-1}$
Substitute using the facts that $\Sigma X = n\bar{X}$ and $\Sigma 1 = n$.	$s^2 = \dfrac{(\Sigma X^2) - 2\bar{X} \cdot (n\bar{X}) + \bar{X}^2 \cdot n}{n-1}$
Simplify the last two terms in the numerator.	$s^2 = \dfrac{(\Sigma X^2) - 2n\bar{X}^2 + n\bar{X}^2}{n-1}$
Combine the last two terms in the numerator.	$s^2 = \dfrac{(\Sigma X^2) - n\bar{X}^2}{n-1}$
Multiply numerator and denominator by n.	$s^2 = \dfrac{n(\Sigma X^2) - n^2\bar{X}^2}{n(n-1)}$
Modify the second term in the numerator.	$s^2 = \dfrac{n(\Sigma X^2) - (n\bar{X})^2}{n(n-1)}$
Substitute using the fact that $n\bar{X} = \Sigma X$.	$s^2 = \dfrac{n(\Sigma X^2) - (\Sigma X)^2}{n(n-1)}$

Thus $s^2 = \dfrac{\Sigma(X - \bar{X})^2}{n-1} = \dfrac{n(\Sigma X^2) - (\Sigma X)^2}{n(n-1)}$.

by Patrick Quigley

Deriving the Shortcut Formula for the Population Variance

Steps	Result
Given.	$\sigma^2 = \dfrac{\Sigma(X-\mu)^2}{N}$
Square the binomial in the numerator.	$\sigma^2 = \dfrac{\Sigma(X^2 - 2\mu \cdot X + \mu^2)}{N}$
Split up the sum.	$\sigma^2 = \dfrac{(\Sigma X^2) - \Sigma(2\mu \cdot X) + \Sigma(\mu^2)}{N}$
Factor out the constants 2 and μ.	$\sigma^2 = \dfrac{(\Sigma X^2) - 2\mu \cdot (\Sigma X) + \mu^2 \cdot (\Sigma 1)}{N}$
Substitute using the facts that $\Sigma X = N\mu$ and $\Sigma 1 = N$.	$\sigma^2 = \dfrac{(\Sigma X^2) - 2\mu \cdot (N\mu) + \mu^2 \cdot N}{N}$
Simplify the last two terms in the numerator.	$\sigma^2 = \dfrac{(\Sigma X^2) - 2N\mu^2 + N\mu^2}{N}$
Combine the last two terms in the numerator.	$\sigma^2 = \dfrac{(\Sigma X^2) - N\mu^2}{N}$
Multiply numerator and denominator by *N*.	$\sigma^2 = \dfrac{N(\Sigma X^2) - N^2\mu^2}{N^2}$
Modify the second term in the numerator.	$\sigma^2 = \dfrac{N(\Sigma X^2) - (N\mu)^2}{N^2}$
Substitute using the fact that $N\mu = \Sigma X$.	$\sigma^2 = \dfrac{N(\Sigma X^2) - (\Sigma X)^2}{N^2}$

Thus $\sigma^2 = \dfrac{\Sigma(X-\mu)^2}{N} = \dfrac{N(\Sigma X^2) - (\Sigma X)^2}{N^2}$.

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