Math 10: Exam 2 (Fall 2009) **Solutions**

You will not receive full credit if you do not clearly show work as demonstrated in class. Show all work in the space provided on this exam. Circle your answers.

1. The probability that a high school student is taking both calculus and physics is 3%. The probability that a student is taking calculus is 5%. Find the probability that a student is taking physics given that he or she is taking calculus. (8 points)

$$P(\text{physics} \mid \text{calculus}) = \frac{P(\text{physics and calculus})}{P(\text{calculus})} = \frac{0.03}{0.05} = 0.06$$

2. How many different committees of five can be selected from a class of 45 students? (8 points)

$$_{45}C_5 = \frac{45!}{(45-5)!(5!)} = \frac{45!}{(40!)(5!)} = \frac{45\cdot 44\cdot 43\cdot 42\cdot 41}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 1{,}221{,}759$$

3. How many different three digit numbers can be constructed from the digits of the number 86,243 if the digits are selected without replacement? (8 points)

$$_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{2\cdot 1} = 60$$

4. A single card is selected at random from a regular deck of 52 cards. (22 points)

a. Find P(the card is a king)

There are four kings so…

$$P(\text{the card is a king}) = \frac{4}{52} = \frac{1}{13}$$

b. Find P(the card is a diamond)

There are thirteen diamonds so…

$$P(\text{the card is a diamond}) = \frac{13}{52} = \frac{1}{4}$$

c. Find P(the card is not a diamond)

$$P(\text{the card is a not diamond}) = 1 - P(\text{the card is a diamond}) = 1 - \frac{1}{4} = \frac{3}{4}$$

d. Find P(the card is a king and the card is not a diamond)

There are three kings which are not diamonds so…

$$P(\text{the card is a king and the card is not a diamond}) = \frac{3}{52}$$

e. Find P(the card is a king or the card is not a diamond)

$$\begin{aligned} P(\text{king or not a diamond}) &= P(\text{king}) + P(\text{not a diamond}) - P(\text{king and not a diamond}) \\ &= \frac{1}{13} + \frac{3}{4} - \frac{3}{52} \\ &= \frac{4}{52} + \frac{39}{52} - \frac{3}{52} \\ &= \frac{40}{52} \\ &= \frac{10}{13} \end{aligned}$$

5. In a particular town, the probability that a person owns a car is 0.87, that a person owns a boat is 0.17, and that a person owns both a car and a boat is 0.10. Find the probability that a randomly selected person owns a boat or a car. (8 points)

$$P(\text{boat or car}) = P(\text{boat}) + P(\text{car}) - P(\text{boat and car}) = 0.17 + 0.87 - 0.10 = 0.94$$

6. Approximately 17% of the adult U.S. population consists of senior citizens (65 years or older). If a jury of twelve adults is randomly selected from this population, what is the probability that the jury will include at least one senior citizen? (8 points)

For a single selection:

$$P(\text{not a senior citizen}) = 1 - P(\text{senior citizen}) = 1 - 0.17 = 0.83$$

For twelve selections:

$$P(\text{at least one senior citizen}) = 1 - P(\text{no senior citizens}) = 1 - (0.83)^{12} = 1 - 0.107 = 0.893$$

7. Complete each formula below for n distinct items taken r at a time. (8 points)

a. $${}_nC_r = \frac{n!}{(n-r)!(r!)}$$

b. $${}_nP_r = \frac{n!}{(n-r)!}$$

8. A child randomly selects three candies (without replacement) from a jar containing 5 grape, 4 orange, and 8 cherry candies. Find the probability that she selects: (20 points)

a. two cherry candies and one grape

We select one of the five grape, none of the four orange and two of the eight cherry, so…

$$P(\text{two cherry candies and one grape}) = \frac{{}_5C_1 \cdot {}_4C_0 \cdot {}_8C_2}{{}_{17}C_3} = \frac{5 \cdot 1 \cdot 28}{680} = \frac{140}{680} = \frac{7}{34}$$

b. one candy of each flavor

We select one of the five grape, one of the four orange and one of the eight cherry, so…

$$P(\text{one candy of each flavor}) = \frac{{}_5C_1 \cdot {}_4C_1 \cdot {}_8C_1}{{}_{17}C_3} = \frac{5 \cdot 4 \cdot 8}{680} = \frac{160}{680} = \frac{4}{17}$$

c. two grape candies and one cherry

We select two of the five grape, none of the four orange and one of the eight cherry, so…

$$P(\text{two grape candies and one cherry}) = \frac{{}_5C_2 \cdot {}_4C_0 \cdot {}_8C_1}{{}_{17}C_3} = \frac{10 \cdot 1 \cdot 8}{680} = \frac{80}{680} = \frac{2}{17}$$

d. three orange candies

We select none of the five grape, three of the four orange and none of the eight cherry, so…

$$P(\text{three orange candies}) = \frac{{}_5C_0 \cdot {}_4C_3 \cdot {}_8C_0}{{}_{17}C_3} = \frac{1 \cdot 4 \cdot 1}{680} = \frac{4}{680} = \frac{1}{170}$$

9. Fifteen percent of the employees at a particular company are in management positions, and sixty percent of the managers are female. Seventy-five percent of non-management positions are filled by male employees. If an employee is selected at random, what is the probability that the selected person is male? (10 points)

A male employee can be either a manager or not a manager, so…

$$\begin{aligned} P(\text{male}) &= P(\text{male and manager}) + P(\text{male and not manager}) \\ &= P(\text{manager}) \cdot P(\text{male} \mid \text{manager}) + P(\text{not a manager}) \cdot P(\text{male} \mid \text{not a manager}) \\ &= (0.15)(0.4) + (0.85)(0.75) \\ &= 0.06 + 0.6375 \\ &= 0.6975 \end{aligned}$$

Formulas

$P(E) + P(\bar{E}) = 1$ $\quad$ $P(E) = 1 - P(\bar{E})$ $\quad$ $P(\bar{E}) = 1 - P(E)$

$P(E) = \dfrac{n(E)}{n(S)}$ $\quad$ $P(B \mid A) = \dfrac{P(A \text{ and } B)}{P(A)}$ $\quad$ $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

$P(A \text{ or } B) = P(A) + P(B)$ $\quad$ $P(A \text{ and } B) = P(A) \cdot P(B)$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$