Math 10: Exam 3 **Solutions**

You will not receive full credit if you do not clearly show work as demonstrated in class. Show all work in the space provided on this exam. Circle your answers.

1. A person pays \$3 to randomly select a single card from an ordinary deck of fifty two. (34 points)
The player wins \$12.00 for a King or Queen, \$9.00 for a Jack, and \$6.00 for an Ace.
The player loses \$13.00 for drawing a 2 or a 3. For any other card, \$0.00 is won.

a. Construct the probability distribution for this game in table form.
Express all probabilities as reduced fractions.

X	\$9		\$6		\$3		-\$16		-\$3		
$P(X)$	2/13		1/13		1/13		2/13		7/13		
$X \cdot P(X)$	(18/13)	+	(6/13)	+	(3/13)	+	(-32/13)	+	(-21/13)	=	-26/13
$X^2 \cdot P(X)$	(162/13)	+	(36/13)	+	(9/13)	+	(512/13)	+	(63/13)	=	782/13

b. Find the mean, variance, and standard deviation for this game.

$$\mu = \sum X \cdot P(X) = -\frac{26}{13} = -\$2$$

$$\sigma^2 = \left[\sum X^2 \cdot P(X)\right] - \mu^2 = \left[\frac{782}{13}\right] - (-2)^2 = \frac{730}{13} = 56.154$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{56.154} \approx \$7.49$$

c. Is this game *fair*? Briefly explain your answer.

No, because $\mu \neq 0$.

2. SAT scores have a mean of 1019 points. If we assume a normal distribution with a standard deviation of 90, find the probability that a randomly selected test score is... (36 points)

a. less than 1181 points.

$$P(X < 1151) =$$
$$P\left(\frac{X-\mu}{\sigma} < \frac{1151-1019}{90}\right) =$$
$$P(z < 1.8) =$$
$$0.9641$$

b. less than 816 points.

$$P(X < 816) =$$
$$P\left(\frac{X-\mu}{\sigma} < \frac{816-1019}{90}\right) =$$
$$P(z < -2.26) =$$
$$0.0119$$

c. between 1077 and 1199 points.

$$P(1077 < X < 1199) =$$
$$P\left(\frac{1077-1019}{90} < \frac{X-\mu}{\sigma} < \frac{1199-1019}{90}\right) =$$
$$P(0.64 < z < 2) =$$
$$0.9772 - 0.7389 =$$
$$0.2383$$

d. between 884 and 1089 points.

$$P(884 < X < 1089) =$$
$$P\left(\frac{884-1019}{90} < \frac{X-\mu}{\sigma} < \frac{1089-1019}{90}\right) =$$
$$P(-1.5 < z < 0.78) =$$
$$0.7823 - 0.0668 =$$
$$0.7155$$

3. The probability of winning on one play of a certain slot machine is 8%. (30 points)
A person plays this slot machine thirty times.

a. Find the mean, variance, and standard deviation of the number of winning plays.

S = the person wins a game $p = 0.08$
F = the person loses a game $q = 0.92$
$n = 30$

$$\mu = np = 30(0.08) = 2.4$$
$$\sigma^2 = npq = 30(0.08)(0.92) = 2.2080$$
$$\sigma = \sqrt{npq} = \sqrt{2.2080} = 1.4854 \approx 1.5$$

b. Find the probability that this person will win more than two of the thirty games.

$$\begin{aligned} P(\text{more than 2 successes}) &= P(X > 2) \\ &= 1 - P(X \le 2) \\ &= 1 - \left[P(X=0) + P(X=1) + P(X=2)\right] \\ &= 1 - \left[\frac{30!}{(30!)(0!)} p^0 q^{30} + \frac{30!}{(29!)(1!)} p^1 q^{29} + \frac{30!}{(28!)(2!)} p^2 q^{28}\right] \\ &= 1 - \left[1(0.08)^0(0.92)^{30} + 30(0.08)^1(0.92)^{29} + 435(0.08)^2(0.92)^{28}\right] \\ &= 1 - \left[0.0820 + 0.2138 + 0.2696\right] \\ &= 1 - 0.5654 \\ &= 0.4346 \end{aligned}$$

Formulas

$\sigma^2 = \sum\left[X^2 \cdot P(X)\right] - \mu^2$ $z = \dfrac{X - \mu}{\sigma}$ $\sigma^2 = npq$

$X = \mu + z\sigma$ $P(X) = \dfrac{n!}{(n-X)!X!} \cdot p^X \cdot q^{n-X}$ $\mu = np$ $\mu = \sum\left[X \cdot P(X)\right]$