Math 10: Exam 4 (Fall 2009) **Solutions**

You will not receive full credit if you do not clearly show work as demonstrated in class. Show all work in the space provided on this exam. Circle your answers.

1. How large a sample should be surveyed to estimate the true proportion of college students who do laundry once a week within 3% with 95% confidence? (12 points)

$$\begin{cases} \hat{p} = 0.5 \\ \hat{q} = 0.5 \\ E = 0.03 \\ z_{\alpha/2} = 1.96 \end{cases}$$

$$\begin{aligned} n &= \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2 \\ &= (0.5)(0.5)\left(\frac{1.96}{0.03}\right)^2 \\ &= 1067.\overline{1} \\ &\approx 1068 \end{aligned}$$

2. The average electric bill in a residential area is \$72 for the month of April. The standard deviation is \$6. If the amounts of the electric bills are normally distributed, find the probability that the mean expense for a random sample of 15 residents will be greater than \$69. (20 points)

$$\begin{cases} n = 15 \\ \mu = 72 \\ \sigma = 6 \end{cases}$$

$$\begin{aligned} P(\overline{X} > 69) &= P\left(\frac{\overline{X} - \mu}{(\sigma / \sqrt{n})} > \frac{69 - 72}{(6 / \sqrt{15})}\right) \\ &= P(z > -1.94) \\ &= 1 - 0.0262 \\ &= 0.9738 \end{aligned}$$

3. The average weight of 40 randomly selected minivans is 4150 pounds. The population standard deviation is 480 pounds. Find the 99% confidence interval of the true mean weight of the minivans. (20 points)

$$\begin{cases} n = 40 \\ \bar{X} = 4150 \\ \sigma = 480 \\ z_{\alpha/2} = 2.58 \end{cases}$$

$$\begin{aligned} E &= z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \\ &= 2.58\left(\frac{480}{\sqrt{40}}\right) \\ &= 195.8 \\ &\approx 196 \end{aligned}$$

$$\begin{gathered} \bar{X} - E < \mu < \bar{X} + E \\ 4150 - 196 < \mu < 4150 + 196 \\ 3954 < \mu < 4346 \end{gathered}$$

4. The mean heart rate for a random sample of 5 students taking a final exam was 96 beats per minute, and the sample standard deviation was 8 beats per minute. Find the 90% confidence interval of the true mean. (20 points)

$$\begin{cases} n = 5 \\ \bar{X} = 96 \\ s = 8 \\ d.f. = 4 \\ t_{\alpha/2} = 2.132 \end{cases}$$

$$\begin{aligned} E &= t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \\ &= 2.132\left(\frac{8}{\sqrt{5}}\right) \\ &= 7.63 \\ &\approx 8 \end{aligned}$$

$$\begin{gathered} \bar{X} - E < \mu < \bar{X} + E \\ 96 - 8 < \mu < 96 + 8 \\ 88 < \mu < 104 \end{gathered}$$

5. In a large university, 30% of the incoming freshmen elect to enroll in a personal finance course offered by the university. Find the probability that of 800 randomly selected incoming freshmen, at least 260 have elected to enroll in the course. (28 points)
Use the normal approximation to the binomial distribution.

$$\begin{cases} n = 800 \\ p = 0.3 \\ q = 0.7 \end{cases}$$

$$\mu = np = 800(0.3) = 240$$
$$\sigma = \sqrt{npq} = \sqrt{800(0.3)(0.7)} = \sqrt{168} \approx 12.96$$

Since $np = 240 \geq 5$ and $nq = 560 \geq 5$, we can use the normal approximation to the binomial distribution.

To approximate $P(X \geq 260)$ for this binomial we need to evaluate $P(X > 259.5)$ for a normal distribution with the same mean and standard deviation as the binomial distribution.

$$\begin{aligned} P(X > 259.5) &= P\left(\frac{\bar{X} - \mu}{\sigma} > \frac{259.5 - 240}{12.96}\right) \\ &= P(z > 1.5) \\ &= 1 - 0.9332 \\ &= 0.0668 \end{aligned}$$

Formulas

$\bar{X} - E < \mu < \bar{X} + E$	$E = z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$	$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$	$\mu = np$	$\sigma = \sqrt{npq}$
$\bar{X} - E < \mu < \bar{X} + E$	$E = t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$	$d.f. = n - 1$	$z = \frac{X - \mu}{\sigma}$	$z = \frac{\bar{X} - \mu}{(\sigma / \sqrt{n})}$
$\hat{p} - E < p < \hat{p} + E$	$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$	$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$	$\hat{p} = \frac{X}{n}$	$\hat{q} = 1 - \hat{p} = \frac{n - X}{n}$